CHAPTER 17
ENGINEERING COST ANALYSIS

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17.1 INTRODUCTION

In the early 1970s, life cycle costing (LCC) was adopted by the federal government. LCC is a method of evaluating all the costs associated with acquisition, construction and operation of a project. LCC was designed to minimize costs of major projects, not only in consideration of acquisition and construction, but especially to emphasize the reduction of operation and maintenance costs during the project life.

Authors of engineering economics texts have been very reluctant and painfully slow to explain and deal with LCC. Many authors devote less than one page to the subject. The reason for this is that LCC has several major drawbacks. The first of these is that costs over the life of the project must be estimated based on some forecast, and forecasts have proven to be highly variable and frequently inaccurate. The second problem with LCC is that some life span must be selected over which to evaluate the project, and many projects, especially renewable energy projects, are expected to have an unlimited life (they are expected to live forever). **The longer the life cycle, the more inaccurate annual costs become** because of the inability to forecast accurately.

This chapter on engineering cost analysis is designed to provide a basic understanding and the elementary skills to complete a preliminary LCC analysis of a proposed project. The time value of money is discussed and mathematical formulas for dealing with the cash flows of a project are derived. Methods of cost comparison are presented. Depreciation methods and depletion allowances are included combined with their effect on the after-tax cash flows. The computer program RELCOST, designed to perform LCC for renewable energy projects, is also presented. A discussion of caveats related to performing LCC is included. No one should attempt to do a comprehensive cost analysis of any project without an extensive background on the subject, and considerable expertise in the current tax law.

17.1.1 Use of Interest Tables

When performing engineering cost analysis, it is necessary to apply the mathematical formulas developed in this chapter and avoid using interest tables for the following reasons:

1. Interest rates applying to real world problems are not found in interest tables, and therefore, interpolation is required.

   When trying to solve problems with interpolation the assumption is made that compound interest formulas are linear functions. THEY ARE NOT. They are logarithmic functions.

2. Not only are real world interest rates difficult to find in tables, but it is frequently difficult to find the required number of interest periods for the project in a set of tables.

3. If the need arises to convert a frequently compounded interest rate to a weekly or monthly interest rate, it is almost certain the value of the effective interest rate will not be in any interest table.

4. Renewable energy projects, especially those for district heating systems, can run into hundreds of millions of dollars. Although it is understood that this chapter was written for preliminary economic studies, nevertheless, interpolation of interest tables can cause an error many times larger than the cost analyst's annual salary.

5. With today's microcomputers and sophisticated hand-held calculators, interest tables are obsolete. Calculators capable of computing all time value functions except gradients are available for under $20. These calculators can also solve the number of interest periods and iterate an interest rate to nine decimal places.

17.2 THE TIME VALUE OF MONEY

The concept of the time value of money is as old as money itself. Money is an asset, the same as plant and equipment and other owned resources. If equipment is borrowed, a plant is rented or land is leased, the owner should receive equitable compensation for its use. If money is borrowed, the lender should be reimbursed for its use. The rent paid for using someone else's money is called interest. Interest takes two different forms: simple interest and compound interest.

Throughout this chapter, the time value of money and compound interest are used in the cost analysis of projects. Such things as risk and uncertainty are ignored, and the concept of an unstable dollar or the value of the dollar fluctuating in the foreign market are not considered. However, in LCC analysis of renewable energy projects, inflation rates for operation and maintenance, equipment purchases, energy consumed and the revenue from energy sold for both conventional and renewable energy, will be considered.

The concept of the time value of money evolves from the fact that a dollar today is worth considerably more than a promise to pay a dollar at some future date. The reason this is true is because a dollar today could be invested and be earning interest such that, at sometime in the future, the interest earned would make the investment worth considerably more than one dollar. To illustrate the time value of money, it is convenient to consider money invested at a simple interest rate.

17.2.1 Simple Interest

Simple interest is interest accumulated periodically on a principal sum of money that is provided as a loan or invested at some rate of interest (i), where i represents an interest rate per interest period. It is important to notice that in problems involving simple interest, interest is only charged or earned on the original amount borrowed or invested. Consider a deposit of $100 made into an account that pays 6% simple interest annually. If the money is left on deposit for 1 year, the balance at the end of year one would be:

$$100 + 0.06(100) = \$106.$$

If the money is left on deposit for 2 years, the balance at the end of year two would be:

$$100 + 0.06(100) + 0.06(100) = \$112.$$

If the money is left on deposit for 3 years, the balance at the end of year three would be:

$$100 + 0.06(100) + 0.06(100) + 0.06(100) = \$118.$$

If n equals the number of interest periods the money is left on deposit and i equals the rate of interest for each period, the formula for calculating the balance at the end of n periods would be:

$$100 + 100(i \times n).$$

Substituting 6% for i and 3 for n, the formula becomes:

$$100 + 100(0.06 \times 3) = \$118.$$

Substituting present value (Pv) for the amount of money loaned or deposited at time zero (beginning of the time period covered by the investment), and future value (Fv) for the balance in the account at the end of n periods, the formula becomes:

$$Fv = Pv + Pv(i \times n).$$

Factoring out Pv, the formula becomes:

$$Fv = Pv(1 + i \times n). \qquad (17.1)$$

Going back to the original values, if the $100 is left on deposit for 5 years, the future value would be $130, and is written:

$$Pv = 100;\ i = 0.06;\ n = 5$$
$$Fv = 100(1 + 0.06 \times 5)$$
$$Fv = \$130.$$

Remember, in simple interest problems, interest is earned only on the amount of the original deposit. Consider the case where interest is calculated more frequently than once per year. This would not change the amount of money earned in simple interest calculations.

Suppose that $100 was deposited for 5 years at a rate of 6% simple interest, calculated every three months. Since there are four 3-month periods in a year, the simple interest per interest period becomes 0.06/4 = 0.015 and n becomes 4 quarters per year x 5 y = 20 total interest periods. Therefore:

$$Fv = 100(1 + 0.015 \times 20)$$
$$Fv = \$130.$$

Applying this formula to the time value of money, it can be shown that for any given rate of interest, $100 received today would be much greater value than $100 received 5 years from today. Consider:

Proposal 1: A promise to pay $100 5 years from today.
Proposal 2: A promise to pay $100 today.

If proposal 2 is accepted over proposal 1, the $100 received today could be deposited into an account that earned 9% annually, and in 5 years the balance would be $145. Using this same theory, the present value of a promise to pay $100 5 years from today can be evaluated as:

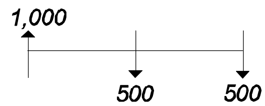
$$Fv = 100;\ i = 0.09;\ n = 5\ y$$
$$100 = Pv(1 + 0.09 \times 5).$$

Solving for Pv, the equation becomes:

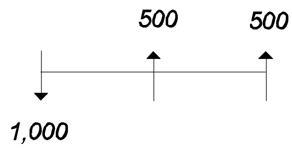
$$Pv = 100/(1 + 0.09 \times 5)$$
$$Pv = \$68.97.$$

Throughout this chapter and in cost analysis texts in general, cash flow diagrams are normally drawn to illustrate monies flowing into or out of a project at some specific time period. The accepted convention is: a) money flowing out is indicated by a down arrow and b) money flowing in is indicated by an up arrow.

Example 17.1: A $1,000 loan to be repaid in two equal annual payments, from the borrower's point of view, would be drawn as:

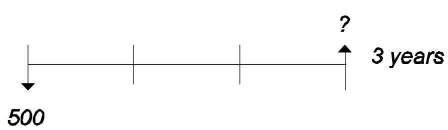


and, from the lender's point of view, would be drawn as:



The examples below illustrate the application of cash flow diagrams.

Example 17.2: A woman deposited $500 for 3 years at 7% simple interest per annum. How much money can be withdrawn from the account at the end of the 3-year period? The cash flow diagram below indicates money deposited into the investment as ↓, and money withdrawn from the investment as ↑,



solution:

Fv = Pv(1 + i x n)
Fv = 500(1 + 0.07 x 3)
Fv = 500(1.21)
Fv = $605.

Example 17.3: Assume $500 is deposited for 200 days in an account that earns 6% simple interest per annum. What is the balance at the end of the investment period? The solution is:

Fv = Pv(1 + i x n)
Fv = 500[1 + 0.06(200/365)]
Fv = 500(1.0329)
Fv = $516.45.

17.2.2 Compound Interest

All compound interest formulas developed will include the standard functional notation for those formulas to the right of the developed formula. Functional notation is a shorthand method of representing a formula to be applied to a problem or a portion of a problem, rather than having to write the formula in its entirety.

For example, (F/P, i, n) is read, "To find the future value F, given the present value, P at an interest rate per period i for n interest periods." This notation applies only to compound interest.

Compound interest varies from simple interest in that interest is earned on the interest accumulated in the account. To illustrate:

If $100 is deposited at 6% compound annually, at the end of the first year the balance would be:

100(1 + 0.06) = $106.

This is the same as in simple interest. However, if the money is allowed to remain on deposit for 2 years, the interest earned during the second year would be:

106(0.06) = $6.36

giving a balance of $112.36 at the end of the second year. If the money is left on deposit for 3 years, the interest earned during the third year would be:

112.36(0.06) = $6.74.

Thus, the balance at the end of the third year would be:

100 + 6 + 6.36 + 6.74 = $119.10.

The mathematical function of compound interest for a deposit of $100 earning 6% compounded annually left on deposit for 3 years is stated and described mathematically below.

Original deposit plus interest earned at the end of the first year becomes:

Fv = 100(1 + 0.06)

plus the interest earned during the second year:

+ 0.06[100(1 + 0.06)]

plus the interest earned during the third year:

+0.06{100(1 + 0.06) + 0.06[100(1 + 0.06)]}.

The formula becomes rather complex with only a 3-year investment. The formula can be simplified through mathematical manipulation. For purposes of this illustration, let 0.06 = i and the number of interest periods = 3, then:

Fv = 100(1 + i) + i[100(1 + i)] + i{100(1 + i) + i [100(1 + i)]}.

Factoring out \$100 from the above equation:

$$Fv = 100[(1 + i) + i(1 + i) + i\{(1 + i) + i(1 + i)\}]$$

simplifying:

$$Fv = 100[1 + i + i + i^2 + i(1 + i + i + i^2)]$$

simplifying further:

$$Fv = 100(1 + i + i + i^2 + i + i^2 + i^2 + i^3)$$

and collecting terms:

$$Fv = 100(1 + 3i + 3i^2 + i^3)$$

then, this equation can be factored into:

$$Fv = 100[(1 + i)(1 + i)(1 + i)] = 100(1 + i)^3.$$

Substituting n for the number of interest periods, which in this case is 3, the result is:

$$Fv = 100(1 + i)^n.$$

Letting Pv = the amount of the investment, then:

$$Fv = Pv(1 + i)^n \qquad (F/P,i,n)(17.2)$$

This is the **single payment compound amount factor**.

Solving Equation (17.2) for Pv gives:

$$Pv = \frac{Fv}{(1 + i)^n} \qquad (P/F,i,n)(17.3)$$

which is the **single payment present worth factor**.

With the development of the equation for finding the future value of a lump sum investment at a compound interest rate for n interest periods, it can be shown how more frequent compounding increases the interest earned.

An interest rate of 3%/6 mo would be stated in nominal form as 6% compounded semiannually.

Consider the following examples with interest rates stated as an annual percentage rate (APR), commonly referred to as the "nominal interest rate."

Example 17.4: An amount of \$100 is invested for 3 years in an account that earns 18% compounded annually. The future value at the end of the 3-year period will be:

$$Fv = Pv(1 + i)^n$$
$$Fv = 100(1 + 0.18)^3$$
$$Fv = 100(1.6430)$$
$$Fv = \$164.30.$$

Example 17.5: Assume \$100 is invested for three years in an account that earns 18% compounded quarterly. The future value at the end of the 3-year period is:

$$Fv = Pv(1 + i)^n$$

where

i = 0.18/4 quarters/y = 0.045 per quarter
n = 3 y x 4 quarters/y = 12 interest periods.

Solution:

$$Fv = 100(1 + 0.045)^{12}$$
$$Fv = 100(1.6959)$$
$$Fv = \$169.59.$$

Example 17.6: Suppose \$100 is invested in an account that earns 18% compounded monthly. The future value at the end of a 3-year period is:

$$Fv = Pv(1 + i)^n$$

where

i = 0.18/12 months/y = 0.015/mo
n = 3 y x 12 mo/y = 36 interest periods.

Solution:

$$Fv = 100(1 + 0.015)^{36}$$
$$Fv = 100(1.7091)$$
$$Fv = \$170.91.$$

Example 17.7: An amount of \$100 is invested for 3 years in an account that earns 18% compounded weekly. The future value at the end of the 3-year period is:

$$Fv = Pv(1 + i)^n$$

where

i = 0.18/52 weeks/y = 0.0034615/week
n = 3 y x 52 weeks/y = 156 weeks.

Solution:

$$Fv = 100(1 + 0.0034615)^{156}$$
$$Fv = 100(1.7144)$$
$$Fv = \$171.44.$$

Example 17.8: If \$100 is invested for 3 years in an account that earns 18% compounded daily, the future value at the end of the 3-year period is:

$$Fv = Pv(1 + i)^n$$

where

$i = 0.18/365 \text{ days/y} = 0.00049315/\text{d}$
$n = 3 \text{ y} \times 365 \text{ d/y} = 1095 \text{ d}.$

Solution:

$Fv = 100(1 + 0.00049315)^{1095}$
$Fv = 100(1.71577)$
$Fv = \$171.58.$

Money invested today will grow to a larger amount in the future. If this is true, then the promise to pay some amount of money in the future is worth a smaller amount today.

Example 17.9: What is the present value of a promise to pay $3,000 5 years from today if the interest rate is 12% compounded monthly? This can be written:

$$Pv = \frac{Fv}{(1 + i)^n}$$

where

$i = 0.12/12 = 0.01$
$n = 5 \times 12 = 60.$

Solution:

$Pv = 3{,}000/(1 + 0.01)^{60}$
$Pv = \$1{,}651.35.$

17.2.3 Annual Effective Interest Rates

It is convenient at this point in the development of compound interest to introduce annual effective interest rates. Annual effective interest (AEI) is interest stated in terms of an annual rate compounded yearly, which is the equivalent of a nominally stated interest rate. Table 17.1 illustrates the relationship between nominal interest, interest rate per interest period, and annual effective interest.

Notice that the nominal interest rate remains the same percentage while the compounding periods change. The interest rate per interest period is obtained by dividing the nominal rate by the number of interest periods per year. The annual effective interest rate is the only true indicator of the amount of annual interest, and therefore, annual effective interest provides a true measure for comparing interest rates when the frequency of compounding is different.

The annual effective interest rate may be found for any nominal interest rate as shown below.

Table 17.1 Comparative Interest Rates

(APR) Nominal Interest Rate (18%)	Interest Rate per Interest Period (%)	Annual Effective Interest (AEI) (%)
Compound annually	18.00	18.00
Compounded semiannually	9.00	18.81
Compounded quarterly	4.50	19.25
Compounded monthly	1.50	19.56
Compounded weekly	0.346	19.71
Compounded daily	0.04932	19.716
Compounded continuously		19.7217

Consider a dollar that was invested for 1 year at a nominal rate of 18% compounded monthly. To calculate the balance (Fv) at the end of the year:

$Fv = 1[1 + (0.18/12)]^{12}$

therefore,

$Fv = 1(1.015)^{12}$
$Fv = 1(1.1956)$
$Fv = \$1.1956.$

Because a dollar was invested originally, the annual interest earned may be found by subtracting the original investment:

$1.1956 - 1 = 0.1956$

and the effective interest is 19.56%. Then, the formula for finding annual effective interest is:

$$AEI = \left[1 + \frac{r}{m}\right]^m - 1 \qquad (17.4)$$

where

AEI = annual effective interest rate
r = nominal interest rate/y
m = number of interest periods/y.

The annual percentage rate is divided by the number of compounding periods/y and raised to the power of the number of compounding periods/y, and 1 is subtracted from that answer to arrive at the annual effective interest rate.

The following examples are used to further illustrate the differences between APR and AEI.

Example 17.10: For an APR of 12% compounded annually, the annual effective interest (AEI) is 12%. Therefore:

AEI = 12% compounded annually. There is no difference between the two.

Example 17.11: For an APR of 12% compounded semi-annually, the semiannual effective interest rate is 0.12/2 = 0.06 or 6%, presented as:

$AEI = (1 + 0.12/2)^2 - 1 = 0.1236$ or 12.36%.

Example 17.12: For an APR of 12% compounded quarterly, the quarterly effective interest rate is 0.03 or 3%, AEI becomes:

$AEI = (1 + 0.12/4)^4 - 1 = 0.1255$ or 12.55%.

Example 17.13: For an APR of 12% compounded daily, the daily effective interest rate is 0.12/365 = 0.003288 or 0.3288%, giving:

$AEI = (1 + 0.12/365)^{365} - 1 = 0.12747$ or 12.75%.

When interest is compounded continuously (when n approaches infinity), the annual effective interest rate takes the form of e - 1, where e = the natural logarithm 2.7182818. Therefore, an APR of 12% compounded continuously would yield an AEI of $(2.7182818)^{0.12} - 1 =$ 12.749%.

Effective interest rates can be calculated for periods other than annually.

To find a weekly effective interest rate of 12% compounded daily, the weekly effective interest rate would be:

$(1 + 0.12/365)^7 - 1 = 0.0023$, or 0.23%.

Therefore,

$$EI = \left[1 + \frac{r}{m}\right]^c - 1 \qquad (17.5)$$

where

r = annual percentage rate
m = number of compound periods/year
c = number of compound periods for the time frame of the effective interest rate.

Example 17.14: The present value of a promise to pay \$4,000 6 years from today at an interest rate that is compounded quarterly is \$2,798. Find the nominal interest rate and the annual effective interest rate.

$Fv = Pv(1 + i)^n$

where

n = 6 x 4 = 24 quarters

and solving

$4{,}000 = 2{,}798(1 + i)^{24}$

dividing both sides by 2,798

$1.42959 = (1 + i)^{24}$

taking the 24th root of both sides

$(1.42959)^{0.04167} = [(1 + i)^{24}]^{0.04167}$

$1.015 = 1 + i$

$i = 0.015$, or 1.5%.

By definition, i is the interest rate per interest period. Therefore, the answer, 1.5%, is the interest rate per quarter. In order to find the nominal interest rate (or annual percentage rate), it is necessary to multiply i times the number of quarters per year. The nominal interest rate is 0.015 x 4 = 0.06, or 6% compounded quarterly. The answer would be incorrect if the frequency of compounding was not included. If the nominal rate is given as 6%, this would indicate 6% compounded annually.

The annual effective interest rate, using Equation (17.4), becomes:

$$AEI = \left[1 + \frac{0.06}{4}\right]^4 - 1$$

$AEI = (1 + 0.015)^4 - 1$

$AEI = 0.0614$, or 6.14%.

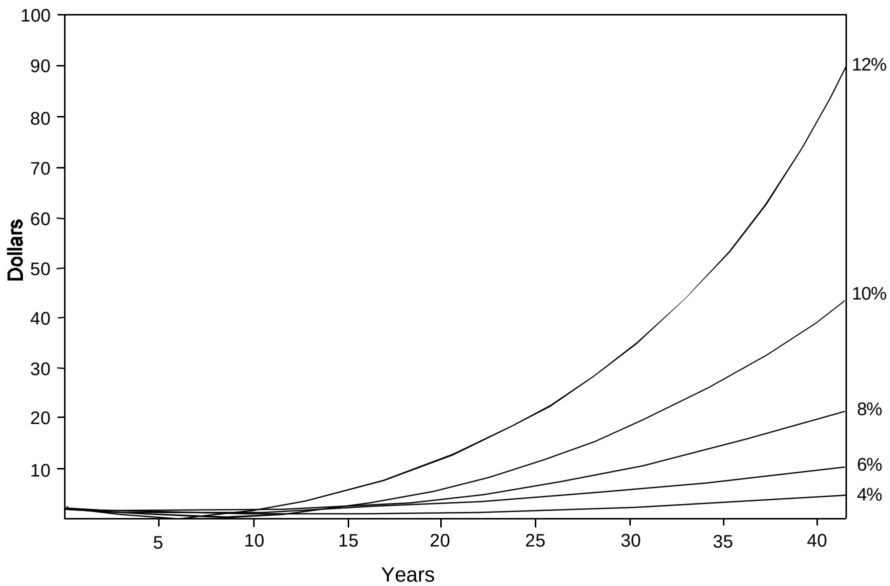


Figure 17.1 Exponential nature of compound interest rates.

The formulas developed for compound interest and the similar formula for converting APR to AEI are **logarithmic functions** (see Figure 17.1).

When interest rates are extremely low, the number of compounding periods is almost insignificant. For example, 2% APR compounded annually is 2% AEI; 2% APR compounded daily is 2.02% AEI while 40% APR compounded daily jumps to 49.15% AEI.

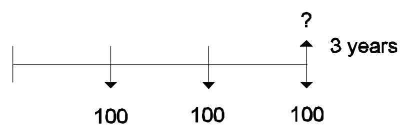
In evaluating projects for nonprofit organizations, interest rates are usually kept rather low, but there are many private entities that evaluate alternatives at the corporation's rate of return, which can be a very high rate.

17.3 ANNUITIES

17.3.1 Ordinary Annuities

The definition of an ordinary annuity is a stream of equal end-of-period payments. Ordinary annuities are the most common form of payment series used in cost analysis. Loan payments, car payments, charge card payments, maintenance and operating costs of equipment are all stated in terms of ordinary annuities. These payments are frequently monthly payments, but they can be weekly, yearly or any other uniform time period. The important thing is that they begin at the end of the first interest period. For example, a person purchases an automobile for $5,000 and is obligated to pay $125 per month for 48 months. The first payment will be due one month after the purchase of the car and the last payment will be due at the end of month 48. In order to derive a mathematical formula to evaluate ordinary annuities, it is convenient to use the future value formula for compound interest Equation (17.2).

Consider three equal end-of-year payments of $100 each.



To find the future value of this payment series at the end of year three, use the future value formula. Notice that the first payment is two interest periods before the end of the project. Therefore, the future value of the first payment is:

$$Fv = Pv(1 + i)^n$$
$$Fv = 100(1 + i)^2.$$

The future value of the second payment, which is one interest period before the end of the project, is:

$$Fv = 100(1 + i)^1.$$

The third payment is zero interest periods away from the end of the project. Therefore, the future value of payment number three is simply $100. The future value of all three payments is:

$$Fv = 100(1 + i)^2 + 100(1 + i)^1 + 100.$$

Reversing the order of these terms and factoring out $100, the equation becomes:

$$Fv = 100[1 + (1 + i)^1 + (1 + i)^2].$$

Notice that with a 3-year project and three annual payments, n does not get higher than 2. This is because the payments are made at the end of each period. Letting A = the amount of each payment, a general equation can be written to find the future value of n payments as:

$$Fv = A[1 + (1 + i)^1 + (1 + i)^2 + (1 + i)^3 + ... + (1 + i)^{n-1}].$$

Multiplying this equation by (1 + i) results in:

$$Fv(1 + i) = A[(1 + i)^1 + (1 + i)^2 + (1 + I)^3 + (1 + i)^4 + ... + (1 + i)^n].$$

Subtracting the first equation from the second equation:

$$Fv(1 + i) - Fv = A[-1 + (1 + i)^n]$$

also

$$Fv + i(Fv) - Fv = A[(1 + i)^n - 1]$$

and

$$i(Fv) = A[(1 + i)^n - 1]$$

dividing both sides of the equation by i gives:

$$Fv = A\left[\frac{(1 + i)^n - 1}{i}\right] \qquad \text{(F/A,i,n)(17.6)}$$

This is the **uniform series compound amount factor**.

Solving for A instead of Fv gives:

$$A = Fv\left[\frac{i}{(1 + i)^n - 1}\right] \qquad \text{(A/F,i,n)(17.7)}$$

This the **uniform series sinking fund factor**.

Returning to the future value formula, $Fv = Pv(1 + i)^n$, and substituting the right side of this equation for Fv in Equation (17.6) gives:

$$Pv(1 + i)^n = A\left[\frac{(1 + i)^n - 1}{i}\right]$$

dividing both sides by $(1 + i)^n$ yields:

$$Pv = A\left[\frac{(1 + i)^n - 1}{i(1 + i)^n}\right] \qquad \text{(P/A,i,n)(17.8)}$$

This is the **uniform series present worth factor**.

Equation (17.8) is used to determine the present value of an ordinary annuity.

Solving the above equation for A instead of Pv gives:

$$A = Pv\left[\frac{i(1 + i)^n}{(1 + i)^n - 1}\right] \qquad \text{(A/P,i,n)(17.9)}$$

This is the **uniform series capital recovery factor**.

Equation (17.9) is used for finding the payment series of a loan or the annual equivalent cost of a purchased piece of equipment.

In calculating future value or present value of a payment series, i must equal the interest rate per payment period, and n must equal the total number of payments. If an interest rate is stated as 12% compounded monthly (APR), and the payment series is monthly, then i = 0.12/12 = 0.01 or 1%/mo.

Complications can arise when the annual percentage rate is stated in such a manner as to be incompatible with the payment period. For example, assume a loan of $500 to be repaid in 26 equal end-of-week payments with an interest rate of 10% compounded daily. Before this problem can be solved, i must be stated in terms of 1 week. Therefore, the weekly effective interest rate, (WEI) must be calculated as:

$$WEI = (1 + 0.10/365)^7 - 1$$
$$WEI = 0.0019, \text{ or } 0.19\% \text{ per week.}$$

When interest is compounded more frequently than the payment, the amount of each payment (A) can be calculated in the above example by:

$$A = 500(A/P,0.19,26)$$
$$A = \$19.73.$$

If the above problem had an interest rate of 10% compounded quarterly, this would present a case where interest is compounded less frequently than the payment period. Therefore, between compounding periods the interest rate is zero. The payment must coincide with the interest rate. Because 26 weeks constitutes 6 months, and the interest rate is compounded quarterly, there are two quarters in a 26-week period. Therefore, to find the interest rate per quarter, find the payment per quarter and divide that payment by 13 weeks to arrive at a payment per week. The interest rate per quarter is:

0.10/4 = 0.025 or 2.5%
A = 500(A/P,2.5,2)
A = $259.41/quarter.

Dividing the above answer by 13 gives $19.95 as the payment per week.

Returning to the problem at the beginning of 17.3.1 Ordinary Annuities, \$5,000 was financed on an automobile for 48 months at an interest rate of 9.24% compounded monthly. Find the monthly payment:

where

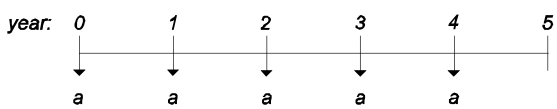
i = 0.0924/12 = 0.0077
n = 4 x 12 = 48.

Solution

A = 5,000(A/P,0.77,48)
A = \$124.996 or \$125/mo.

17.3.2 Annuities Due

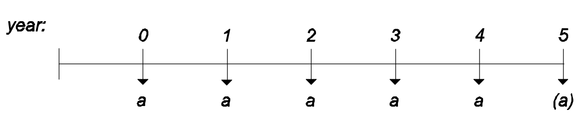
The definition of an annuity due is a **stream of equal, beginning-of-the-period payments**. Although this payment series is not nearly as common as an ordinary annuity, it is still found in many projects. Beginning-of-the-period payments apply to such things as rents, leases, insurance premiums, subscriptions, etc. Rather than attempt to derive a formula to evaluate annuities due, it is much simpler to modify the existing formulas that have already been derived. Consider a 5-year cash flow diagram with payments made at the beginning of each year. These payments will be designated as (a) to avoid confusion with an ordinary annuity. The diagram is shown as:



The easiest approach is to find the future value of this annuity due using a modification of Equation (17.6):

$$Fv = A\left[\frac{(1+i)^n - 1}{i}\right]$$

The cash flow diagram requires modifications as:



By inserting a payment (which does not exist) at the end of year 5, the cash flow diagram now looks like an ordinary annuity of 6 payments. Therefore, using Equation (17.6):

$$Fv = A\left[\frac{(1+i)^6 - 1}{i}\right]$$

However, the sixth payment does not exist. Therefore, that payment, which is zero interest periods away from Fv, must be subtracted from the above formula as:

$$Fv = A\left[\frac{(1+i)^6 - 1}{i} - 1\right]$$

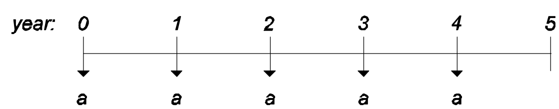
Placing - 1 as the last value inside the brackets subtracts the value of the last payment. Notice that n increased to 6 with only 5 payments; therefore, the general formula for finding the future value of an annuity due is:

$$Fv = a\left[\frac{(1+i)^{n+1} - 1}{i} - 1\right] \qquad \text{(F/a,i,n)(17.10)}$$

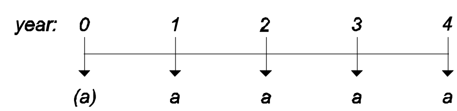
To find the annuity due, given the future value, the formula would be:

$$a = Fv\left[\frac{(1+i)^{n+1} - 1}{i} - 1\right]^{-1} \qquad \text{(a/F,i,n)(17.11)}$$

In order to find the present value of an annuity due of 5 payments use the cash flow diagram:



The present value of this payment series exists at the same point in time as the first payment. Modifying the cash flow diagram, we have:



Ignoring the first payment, the cash flow diagram appears as an ordinary annuity of 4 payments, applying Equation (17.8), we have:

$$Pv = A\left[\frac{(1+i)^4 - 1}{i(1+i)^4}\right]$$

This yields the present value of 4 of the 5 payments but does not consider the first payment, which is at time zero, or the same point as the present value. Therefore, the first payment, which is zero interest periods away from Pv, must be added. By modifying the above equation and placing a + 1 as the last value inside the brackets adds the value of the first payment and gives:

$$Pv = A\left[\frac{(1+i)^4 - 1}{i(1+i)^4} + 1\right]$$

The general formula for finding the present value of an annuity due is:

$$Pv = a\left[\frac{(1+i)^{n-1} - 1}{i(1+i)^{n-1}} + 1\right] \qquad (P/a,i,n)(17.12)$$

To find an annuity due, given the present value, the formula would be:

$$a = Pv\left[\frac{(1+i)^{n-1} - 1}{i(1+i)^{n-1}} + 1\right]^{-1} \qquad (a/P,i,n)(17.13)$$

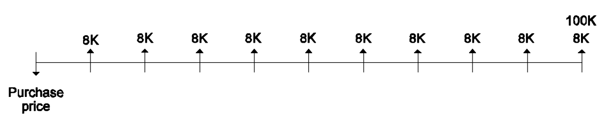
17.3.3 Problems Involving Multiple Functions

Many problems in cost analysis involve the use of several of the formulas presented thus far in this chapter.

Example 17.15: Bonds.

Bonds are sold in order to obtain investment capital. Most bonds pay interest on the face value (the value printed on the bond) either annually or semiannually, and pay back the face value at maturity (the end of the loan period, or end of the life of the bond).

Consider a bond with a face value of $100,000, a life of 10 years, that provides annual interest payments of 8%. How much should be paid for this bond to make it yield 10%? Thus, the cash flow diagram becomes:



The above diagram illustrates the cash flows associated with the bond. The interest is paid out annually to the bond holder. Because there is no opportunity to earn interest on that interest, the bond performs as though it were a problem in simple interest. The performance of the bond cannot deviate from the original cash flow diagram; that is, the bond will always pay $8,000 per year plus $100,000 at maturity. To make this bond pay 10%, the buyer would have to pay less than $100,000. In other words, the bond would have to be sold at a discount price. To calculate the price to pay for the bond to make it yield 10%, the procedure would be:

Price = 8k(P/A,10,10) + 100k(P/F,10,10)
Price = $49,156.54 + $38,554.33
Price = $87,710.87.

To illustrate a premium paid for a bond, assume that the buyer was willing to purchase the bond to yield 7%. Therefore, to make the bond yield lower than 8%, the buyer would have to pay a premium.

Price = 8k(P/A,7,10) + 100k(P/F,7,10)
Price = $56,188.5 + $50,834.93
Price = $107,023.58

Example 17.16: Bank Loans.

A couple financed $50,000 on a home. The terms of the home mortgage were 9.6% compounded monthly for 30 years. After making payments for 5 years, they want to calculate the amount of money they will pay to principal and interest during the 6th year.

Step 1 is to calculate the amount of the monthly loan payment. Using Equation (17.9) gives:

$$A = Pv\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

where

Pv = $50,000
n = 360
i = 0.096/12 = 0.008 or 0.8%
A = 50,000(A/P,0.8,360)
A = $424.08

Now that the payment is calculated, Step 2 is to find the balance of the loan at the end of year 5. Using Equation (17.8) gives:

$$A = Pv\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

where

A = $424.08
i = 0.008
n = 360 - 60 or 300 payments remaining to be made through year 30.
Pv = 424.08(P/A,0.8,300)
Pv = $48,154.90.

Step 3 is to calculate the loan balance at the end of year 6, letting n = 300 - 12 = 288 payments remaining to be made, and using the same process as in Step 2:

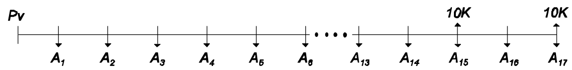
Pv = 424.08(P/A,.8,288)
Pv = $47,667.74.

Subtracting the loan balance, end of year 6, from the loan balance, end of year 5, = $487.16, the amount that will be paid to principal during year 6.

Step 4 is to calculate the amount of the payments that will go to interest. The total amount paid during year 6 will be $424.08 x 12 or $5,088.96. Subtracting the amount that will go to principal, ($487.16), leaves the amount $4,180.86 that will go to interest during year 6.

Example 17.17: Investments.

A couple with two children, ages 2 and 4, want to invest a single annual payment series that will provide \$10,000 to each child at the age of 18. The investment will earn interest at 8.75%. If the annual payments start today and the last deposit is made 16 years from today, what is the amount of each annual payment? To solve this problem, begin by drawing a cash flow diagram as:



The easiest approach is to find the future value of the required monies and then solve for the annuity. Although it is realized that the older child will withdraw \$10,000 14 years from today, the equivalent value of that \$10,000 can be evaluated at the end of the 16th year. Using Equation (17.2) gives:

$Fv = Pv(1 + i)^n$
$Fv = 10,000\ (1 + 0.0875)^2 + 10,000$
$Fv = \$21,826.56.$

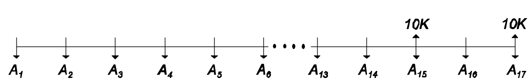
Then, using Equation (17.7), and making n = 17, the amount of each payment is:

$$A = Fv\left[\frac{i}{(1 + i)^n - 1}\right]$$

$$A = 21,826.56\left[\frac{0.0875}{(1 + 0.875)^{17} - 1}\right]$$

A = \$604.00.

If this method is confusing, the problem could have been solved using present value. This requires more calculations and also modification of the cash flow diagram. To approach the problem from a present value point of view, draw a 17-year cash flow diagram as:



In order to solve the problem using present value techniques, it is necessary to note that present value exists 1 year before the first payment. This is the reason for modification of the cash flow diagram. The first \$10,000 payment must be discounted 15 years and the second \$10,000 payment must be discounted 17 years. Using Equation (17.3) we have:

$$Pv = \frac{Fv}{(1 + i)^n}$$

$$Pv = \frac{10,000}{(1 + 0.0875)^{15}} + \frac{10,000}{(1 + 0.0875)^{17}}$$

Pv = \$2,841.59 + \$2,402.71
Pv = \$5,244.30.

This provides the total present value 1 year before the first payment. Now the payment series can be calculated as an ordinary annuity of 17 payments, using Equation (17.9) we have:

$$A = Pv\left[\frac{i(1 + i)^n}{(1 + i)^n - 1}\right]$$

$$A = 5,244.30\left[\frac{0.0875(1 + 0.0875)^{17}}{(1 + 0.0875)^{17} - 1}\right]$$

A = \$604.00.

Notice that, although the final answers are identical, it required much more effort and calculation to solve the problem from a present value point of view.

It is advisable to spend some time evaluating problems, drawing cash flow diagrams and considering the simplest approach. Without a cash flow diagram, it would have been very easy to make the error of assuming there were only 16 payments. It is also doubtful that the problem solver could recognize that the easiest approach for this problem is to work with future value. If there is any doubt regarding the answer, it may be verified by working the problem backwards.

Suppose annual payments of \$604.00 are deposited into an investment for 15 years. The balance in the account, using Equation (17.6) would be \$17,389.40. At this time, the older child withdraws \$10,000, leaving a balance of \$7,389.40. This balance will earn interest for 2 more years, and using Equation (17.2), will amount to \$8,739.12. Meanwhile, two more payments of \$604.00 will be made into the account. These two payments, with interest, will amount to \$1,260.85, using Equation (17.6). The account balance, then, will amount to \$9,999.97. The reason the answer is off by 3 cents is that the actual value of each payment was \$604.00081. Using that value as the payment, the answer would equal exactly \$10,000.

17.4 COST COMPARISON OF INVESTMENT ALTERNATIVES

For the most part, cost analysis involves selection of the minimum cost or maximum profit alternatives. There are basically four accepted methods of evaluating Alternative A as compared to Alternative B as compared to Alternative C.

17.4.1 Present Value Method

The first of these methods is the present value technique, wherein all costs and revenues are discounted back to present value to arrive at a net present value for the project. This method is very time consuming if performed manually and presents problems when the alternatives have different economic lives. Alternative A may have an expected life of 10 years, Alternative B may have an expected life of 15 years, and Alternative C may have an expected life of 20 years. Therefore, in order to do any meaningful evaluation of these three alternatives in terms of present value, it is necessary to find a common denominator for their expected life, which, in this case, would be 60 years, where Alternative A would be replaced six times, B replaced four times, and C replaced three times.

17.4.2 Future Value Technique

The future value technique of evaluating alternatives is almost identical to the present value method except that all costs and revenues are stated in terms of future value. The problem still arises of alternatives with incompatible useful lives. The above techniques are self-explanatory, and because they require such extensive calculations, they will not be covered further. But, because they exist, when evaluating alternatives using computers, it is convenient to indicate, somewhere in the output data generated, the net present value (NPV) of each alternative. Many organizations base their decisions on NPV, and governmental agencies, expect to evaluate benefit-cost ratios. The benefit-cost ratio is the ratio of benefits provided by the alternative versus cost incurred, and will be discussed later in this chapter.

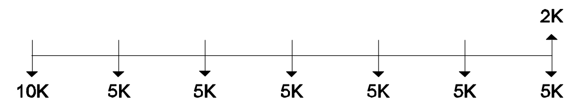
17.4.3 Annual Equivalent Cost Method

The annual equivalent cost method of evaluating alternative projects states all costs and revenues over the useful life of the project in terms of an equal annual payment series (an ordinary annuity). This is probably the most widely used method in the industry, for several reasons:

1. It requires less effort and fewer calculations.
2. It eliminates the problem of alternatives with incompatible useful lives.
3. It allows for much more sophistication when considering inflation, increasing equipment cost, equipment depreciation schedules, etc.

The annual cost method assumes that the project will live forever, and that, if Alternative A has a useful life of 10 years and Alternative B has a useful life of 15 years, each alternative will be replaced at the end of its useful life. Therefore, the alternative with the minimum annual cost or maximum annual profit is the alternative that will be chosen.

Evaluate a piece of equipment that costs $10,000, has a 6-year useful life with a salvage value of $2,000, and annual operating costs of $5,000. The interest rate used for the proposed evaluation is 9% compounded annually. To calculate the cost of purchasing this equipment, operating it for 6 years, and salvaging it at the end of 6 years for $2,000 at 9% interest, the procedure would be to draw a cash flow diagram as:



The typical approach would be to find the annual cost of $10,000, subtract the annual cost of $2,000 salvage, and add the annual operating cost of $5,000. The annual equivalent of the purchase may be found by using Equation (17.9) as:

$$A = Pv \left[\frac{i(1 + i)^n}{(1 + i)^n - 1} \right]$$

$$A = \left[\frac{0.09(1 + 0.09)^6}{(1 + 0.09)^6 - 1} \right] 10K$$

Next, find the annual equivalent cost of the salvage value, using Equation (17.7), as:

$$A = Fv \left[\frac{i}{(1 + i)^n - 1} \right]$$

$$A = - \left[\frac{0.09}{(1 + 0.09)^6 - 1} \right] 2K$$

Finally, add the $5,000 operating cost. The total calculations appear as:

A = $2,229.20 - $265.84 + $5K

$$A = \left[\frac{0.09(1 + 0.09)^6}{(1 + 0.09)^6 - 1} \right] 10K - \left[\frac{0.09}{(1 + 0.09)^6 - 1} \right] 2K + 5K$$

A = $6,963.36.

The value of Equation (17.9), which was used to find the annual equivalent cost of equipment purchased is 0.2229198.

The value of Equation (17.7), which was used to find the annual equivalent cost of the salvage value, is 0.1329198.

Notice that the difference between 0.2229198 and 0.1329198 is exactly equal to the interest rate (i), 0.09. This is true for all interest rates, provided that these functions have the same i and the same n.

In calculating the annual equivalent cost of purchasing the equipment and subtracting the annual equivalent cost of the equipment salvage at some time in the future, this is always the case: i and n are equal; and Equation (17.9) - i = Equation (17.7).

Using functional notation as symbols for these formulas we have:

[(A/P,9,6) - 0.09] = (A/F,9,6).

Making this substitution, the annual cost formula becomes:

A = (A/P,9,6)10K - [(A/P,9,6) - 0.09]2K + 5K

multiplying,

A = (A/P,9,6)10K - (A/P,9,6)2K + 0.09(2K) + 5K

factoring out (A/P,9,6),

A = (A/P,9,6)(10K - 2K) + 0.09(2K) +5K

the general equation for the annual cost equation becomes:

$$A = Pv\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right](cost - slvg) + i(slvg) + OC$$

then

A = (A/P,i,n)(cost - salvage) + i(salvage) + OC (17.14)

represents **the annual equivalent cost formula**

where

OC = annual operating cost.

This modification of the annual cost formula greatly reduces the amount of calculation necessary to arrive at the annual equivalent cost.

Values can be produced by this formula as monthly equivalent costs or weekly equivalent costs by simply changing i to the interest rate per period and allowing n to equal the total number of periods. Using the previous example, suppose the annual percentage rate was 9% compounded monthly. To calculate the periodic costs in terms of monthly equivalent costs (as a monthly ordinary annuity):

A = (A/P,0.75,72)(10K - 2K) + 0.0075(2K) + 5K/12
A = \$144.20 + \$15.00 + \$416.67
A = \$575.87.

Stating costs as an ordinary annuity has nothing to do with actual cost flows. It simply states all costs and revenues as an equal payment series in order that one alternative may be compared with another (See Figure 17.2).

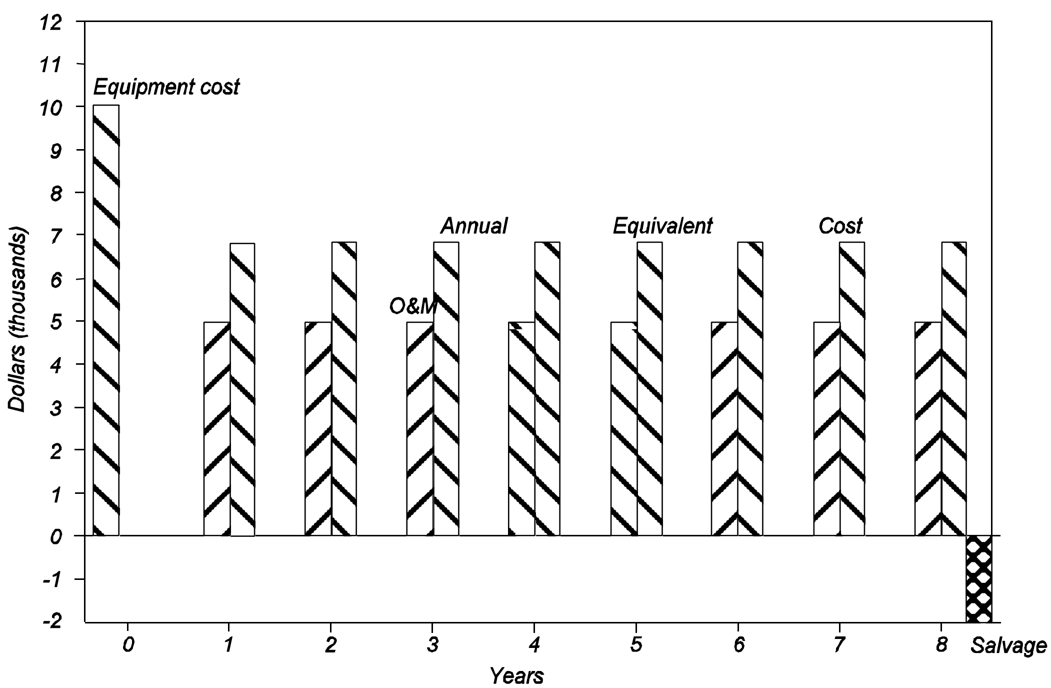


Figure 17.2 Annual Equivalent Cost.

17.4.4 Rate of Return Method

The rate of return (ROR) method of comparing alternatives calculates the interest rate for each alternative and selects the highest ROR. A word of caution is necessary, ROR evaluates INVESTED capital and the costs of operation and maintenance as opposed to revenues or benefits received from the project. Therefore, a project totally financed with borrowed money has no rate of return because there is no invested capital. The following examples assume 100% equity financing and are used to illustrate rate of return calculations.

Example 17.18: An investment of \$7,000 is placed into an account for a 10-year period. At the end of 10 years, the balance in the account is \$17,699.30. What was the annual interest rate earned? Using Equation (17.2):

$Fv = Pv(1 + i)^n$

$\$17,699.30 = \$7,000(1 + i)^{10}$.

This problem can be solved by dividing both sides of the equation by \$7,000 giving:

$17,699.30/7,000 = (1 + i)^{10}$

$2.528 = (1 + i)^{10}$.

Extracting the tenth root of each side of the equation gives:

$(2.528)^{0.1} = [(1 + i)^{10}]^{0.1}$

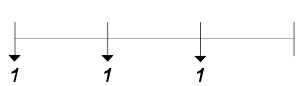
$1.0972 = 1 + i$

$i = 0.0972$.

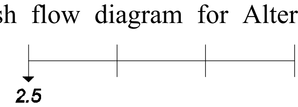
A ROR can also be calculated by evaluating the **difference between alternatives** that provide cost savings rather than revenues.

Example 17.19: Fire insurance premiums on a warehouse are \$500/y (Alternative A). The same coverage can be purchased by paying a 3-year premium of \$1,250 (Alternative B). Find the ROR realized by purchasing a 3-year policy in place of three 1-year policies.

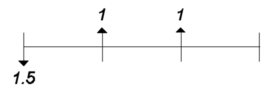
To simplify this problem, notice that the 3-year insurance premium is 2.5 times the annual premium. The cash flow diagram for Alternative A appears below.



The cash flow diagram for Alternative B appears below.



To further simplify this problem, consider the differences between these two alternatives. If Alternative A is subtracted from Alternative B, the cash flow diagram becomes:



Although the insurance premiums are an annuity due, the cash flow diagram above makes the cash flows appear as an investment of \$1.5 providing an ordinary annuity of \$1 at the end of each year for 2 years. The analysis has simplified the problem considerably and avoided using the more complex formulas associated with annuities due. Using Equation (17.8):

$$Pv = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

Entering values in this formula requires that the mathematical signs be properly observed. If money flowing out, below the time line, is considered positive (+) then money flowing in, above the time line, or money saved is considered negative (-). The calculation then becomes:

$$1.5 = -1\left[\frac{(1+i)^2 - 1}{i(1+i)^2}\right]$$

Notice there is one equation and one unknown (i), but the unknown appears three times in the equation. Therefore, the only solution would be by an iterative

$$1.5 = -1\left[\frac{(1+0.21)^2 - 1}{0.21(1+0.21)^2}\right] = -0.00946$$

process. Continuing with Equation (17.8):

where

I = 0.21, or 21%
Pv = 1.5
A = - 1.

The answer to this problem is - 0.00946, indicating that the first interest rate was too low.

Try i = 22%

$$1.5 = -1\left[\frac{(1+0.22)^2 - 1}{0.22(1+0.22)^2}\right] = 0.00847$$

The answer, 0.00847, indicates the interest rate is too high. The actual interest rate is 21.525043%. Placing this interest rate into the formula for i, the answer equals zero, indicating that the ROR realized by purchasing a 3-year policy instead of three 1-year policies is 21.5%. There are many pocket calculators costing under $20 that are designed to calculate financial functions and are capable of iterating for problems involving a single financial function.

Consider the annual cost formula where:

a = (A/P,i,n) x (cost - salvage) + (i) salvage + OC - revenues.

Suppose it is required to find the ROR on such a project. Once again, the solution to the problem is found by iteration. But in this case, there is more than one financial function. Choose an interest rate. Work the problem at that interest rate, and find out if the answer comes out positive or negative. With the mathematical signs we have used in this example, if revenues exceed costs the answer would be negative; if costs exceed revenues the answer would be positive. When the exact i is placed into the formula that denotes the ROR, the answer would be zero. That is, the interest rate that causes revenues and costs to be exactly equal. Interest tables may be used to help find an upper and lower range for i to reduce the number of iterations necessary.

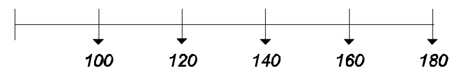
A much easier solution is to place all of the cash flows by year into a computer and write a simple program that can do thousands of iterations in a matter of seconds to find i.

Trying to iterate i for a problem with more than one function on a financial calculator nearly always results in error because of the tremendous amount of time and data that must be entered by hand. Even with a computer program, if the cash flows turn from negative to positive and back to negative during the project life, i can take on the form of a quadratic equation and the computer is unable to determine whether i is positive or negative. There are computer spreadsheets and other software available for iterating i, but these have their limitations. One of the most popular and widely used spread-sheets is designed to iterate i for a series of cash flows. However, this spreadsheet requires a rather accurate guess for i; because, if it does not find i after 20 iterations, the resultant answer is "Err" (error). The RELCOST program developed to accomplish LCC analysis (copyright, Washington State Energy Office) will iterate i to seven decimal places in a matter of seconds for projects with over 150 input variables and more than 500 inflation rates, and evaluate the iterated i to determine whether it is positive or negative. This program will be discussed in more detail in Subsection 17.7, entitled "LIFE CYCLE COST ANALYSIS."

17.5 GRADIENTS

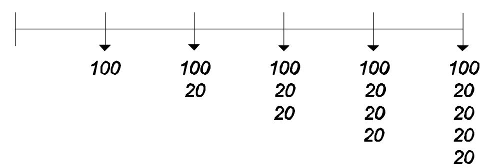
17.5.1 Arithmetic Gradients

Having developed formulas for equal payment series involving both ordinary annuities and annuities due, an end-of-period payment series that increases by a fixed dollar amount at the end of each period can now be evaluated. Consider the following end-of-period payments:



As can be seen in the cash flow diagram above, the payment series started at $100 and increased by $20 per year. Such a payment series is called an arithmetic gradient. Although such a payment series is covered in detail by most texts on engineering economics, it is highly unlikely that any portion of a project will contain such a payment series.

The method used to solve for present value, future value or annual equivalent cost of such a payment series is to break it up into a series of ordinary annuities.



This payment series now appears as five ordinary annuities. Because the series is progressing in the direction of future value, it is easiest to assume one annual payment series of $100 and find the future value of the sum of the remaining four annuities and string them out in the form of an annual equivalent for the 5-year period. The formula to accomplish this, where n = 5 and i = 9%, is:

$$A = 20\left[\frac{1}{0.09} - \frac{5}{(1 + 0.09)^5 - 1}\right]$$

The 5-year equivalent cost of an arithmetic gradient of $20 that begins at the end of year 2 and increases through year 5 is $36.56. Therefore, the annual equivalent of the payment series that began with $100 and increased by $20 for the next 4 years is $100 + $36.56, or $136.56.

Notice that when dealing with an arithmetic gradient, A_1 becomes an ordinary annuity, and the amount of increase starting at the end of the second period is considered to be the gradient. The gradient formula is based on the fact that the gradient always begins at the end of the second period and is to be strung out as an equal payment series from the end of the first period to the end of the cash flow. The general formula for finding the equivalent annual cost of an arithmetic gradient is:

$$A = G\left[\frac{1}{i} - \frac{n}{(1 + i)^n - 1}\right] \qquad \text{(A/G,i,n)(17.15)}$$

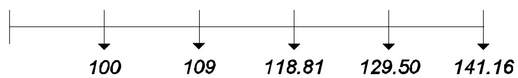
This is the **arithmetic gradient uniform series factor** where G is the amount of increase beginning at the end of the second period.

If the present value or future value of an arithmetic gradient is required, one could simply multiply Equation (17.15) by (P/A,i,n) or (F/A,i,n).

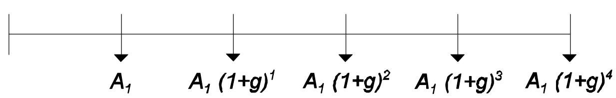
The most common use of arithmetic gradients is in calculating the value of sum-of-years-digits (S-Y-D) depreciation, which takes the form of a negative arithmetic gradient. This method of depreciation is no longer allowed under the 1987 tax law.

17.5.2 Geometric Gradients

A geometric gradient is an end-of-period payment series that increases by a fixed percentage each period. Consider a 5-year payment series that begins with \$100 and increases by 9% every year thereafter. A cash flow diagram used for calculation of the problem is shown as:



The cash flow diagram above illustrates a geometric gradient with the first payment (A_1) equal to \$100 and each subsequent payment increasing by 9% in other words, a \$100 payment inflating at a rate (g) of 9% annually. Then:



and the present value (Pv) for an assumed cost of capital of 12% compounded annually (i) is:

$$Pv = A_1\left[\frac{1}{(1+i)^1}\right] + A_1\left[\frac{(1+g)^1}{(1+i)^2}\right] + A_1\left[\frac{(1+g)^2}{(1+i)^3}\right]$$

$$+ A_1\left[\frac{(1+g)^3}{(1+i)^4}\right] + A_1\left[\frac{(1+g)^4}{(1+i)^5}\right]$$

$$\frac{1}{(1+i)^n} = \textit{the present value factor } (P/F,i,n)$$

Let n equal the number of interest periods (years, in this case). Then:

$$Pv = \sum_{x=1} A_1(1+g)^{x-1}\left[\frac{1}{(1+i)^x}\right]$$

$$Pv = \sum_{x=1} A_1\left[\frac{(1+g)^{x-1}}{(1+i)^x}\right] = \frac{A_1}{(1+i)}\sum_{x=1}\left[\frac{1+g}{1+i}\right]^{x-1}$$

Note that:

$$A_1 + A_1 g + A_1 g^2 + A_1 g^3 + \ldots + A_1 g^{n-1} = A_1\left[\frac{g^n - 1}{g - 1}\right]$$

where g is constant.

Therefore:

$$Pv = \frac{A_1}{(1+i)}\sum_{x=1}\left[\frac{1+g}{1+i}\right]^{x-1} = \frac{A_1}{(1+i)}\left[\frac{\left[\frac{1+g}{1+i}\right]^n - 1}{\left[\frac{1+g}{1+i}\right] - 1}\right].$$

Then, to find the annual equivalent cost (A), considering inflation: annual cost = present cost x capital recovery factor (A/P,i,n) and is written:

A = PV x (A/P,i,n)

$$A = Pv\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

Multiplying by the capital recovery factor:

$$A = \frac{A_1}{(1=i)}\left[\frac{\left[\frac{1+g}{1+i}\right]^n - 1}{\left[\frac{1+g}{1+i}\right] - 1}\right]\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

Simplifying:

$$A = \frac{A_1}{(1+i)}\left[\left[\left[\frac{1+g}{1+i}\right]^n - 1\right]\left[\frac{1+i}{(1+g)-(1+i)}\right]\right]\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

$$A = \frac{A_1}{(g-1)}\left[\left[\frac{1+g}{1+i}\right]^n - 1\right]\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \quad (A/A_1,g,i,n) \quad (17.16)$$

where g does not equal i.

This is the **geometric gradient uniform series factor**, where g does not equal i.

Using functional notation:

$$A = \frac{A_1}{g-i}[(F/P,g,n)(P/F,i,n) - 1](A/P,i,n)$$

Inserting the initial values in this example:

$$A = \frac{1{,}000}{0.09 - 0.12}\,[(1.5386)\ (0.5674)\ - 1](0.2774)$$

(0.2774) = $117.38.

The annual equivalent cost equals $117.38. Figure 17.3, plots a geometric gradient increasing by 9% for 20 years and indicates the annual equivalent cost at a discount rate of 12%.

Notice, by removing the capital recovery factor in Equation (17.16), the equation becomes:

$$Pv = \frac{A_1}{(g - i)}\left[\left[\frac{1 + g}{1 + i}\right]^n - 1\right] \qquad (P/A,g,i,n)(17.17)$$

where g is not = i.

This is the **geometric gradient present worth factor**, where g is not = i.

In the case where the discount rate equals the rate of inflation (g = i), the equation becomes A_1 divided by zero, which is undefined. If a geometric gradient is increasing by exactly the discount rate, this has the same effect of an interest rate equal to zero; therefore, simply take A_1, multiply it by the total number of payments (n), which will yield a present value at the end of year one. Because present value represents the dollar equivalent at time zero, the amount that was calculated by multiplying A_1 by n is one interest period off. To bring it to the proper time frame, time zero, simply discount it by one time period. Therefore:

$$Pv = \frac{A_1(n)}{(1 + i)} \qquad (P/A_1,g,i,n)\ (17.18)$$

where g = i.

This is the **geometric gradient present worth factor**, where g = i.

Although geometric gradients are rather common and are found in many applications, they require that the rate of increase remain constant. Such is not the case in most energy forecasts, where inflation rates are modified year by year. This problem will be discussed later in Section 17.7 under the heading, "LIFE CYCLE COST ANALYSIS."

Figure 17.4 graphs a project with the following input variables:

		20-year Annual Equivalent Costs
Project life in years	20	
Interest rate (APR)	12%	
Capital investment	$120,000	$16,065
Salvage value	12,000	-167
Annual costs		
Insurance	500	560
Fixed	2,250	2,250
Arithmetic gradient	230	1,385
Geometric gradient		
increasing at 10%	2,000	4,051
increasing at 6%	10,000	14,895
Depreciation method	200% declining balance	

Figure 17.4 illustrates the total annual equivalent costs for operating this project for 1 year, 2 years, 3 years, etc. The capital recovery line indicates the annual equivalent of investing capital and salvaging the project at end of year 1, 2, 3, etc. The operation and maintenance line shows the annual equivalent cost of operating the project in years 1 through 20. Notice that the minimum annual cost occurs in year 15, which is $38,486. That is to say, the annual equivalent stream of equal payments for operating the project with a 15-year life would be $38,486/y.

Table 17.2 Economic Life Calculations

Year	Capital Recovery	O&M Costs	Total Costs	Salvage Values
1	$38,400	$14,810	$53,210	$96,000
2	34,777	15,296	50,073	76,800
3	31,754	15,779	47,533	61,440
4	29,224	16,259	45,483	49,152
5	27,100	16,735	43,834	39,322
6	25,311	17,207	42,517	31,457
7	23,800	17,674	41,473	25,166
8	22,519	18,135	40,655	20,133
9	21,431	18,592	40,023	16,106
10	20,554	19,042	39,596	12,000
11	19,629	19,485	39,114	12,000
12	18,875	19,922	38,797	12,000
13	18,253	20,352	38,605	12,000
14	17,734	20,774	38,509	12,000
15	17,297	21,189	38,486	12,000
16	16,926	21,596	38,522	12,000
17	16,609	21,995	38,604	12,000
18	16,337	22,385	38,722	12,000

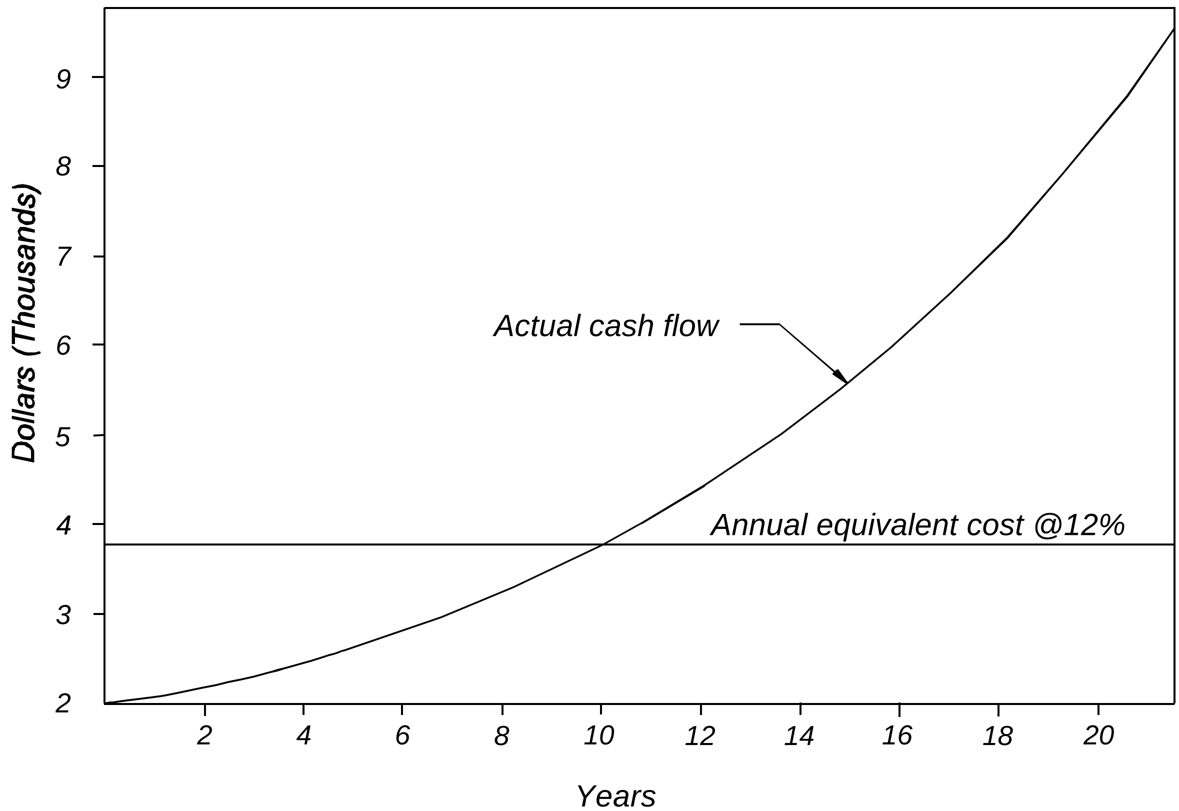


Figure 17.3 Geometric gradient.

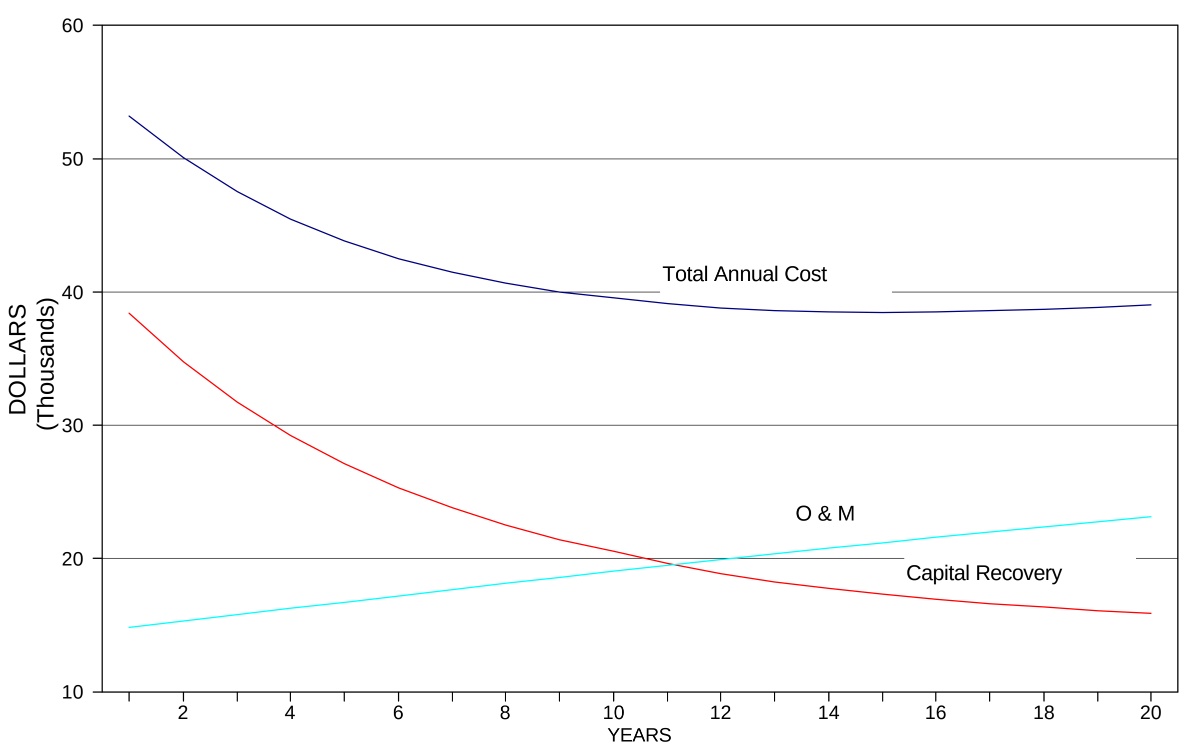


Figure 17.4 Economic life.

Table 17.2 provides annual equivalent values from year 1 through year 18 for capital recovery, operation and maintenance costs, and total annual costs. Salvage values for any given year are indicated in the last column. This analysis is beyond the scope of this chapter, but could be used to determine the economic life (minimum annual cost) of a project that had these cost characteristics. Table 17.2 provides answers for various costs that could be beneficial to those who want to sharpen their expertise in calculating capital recovery, annuities due, ordinary annuities, geometric gradients, and arithmetic gradients.

17.6 EQUIPMENT DEPRECIATION

A discussion of equipment depreciation is essential in evaluating projects for taxable entities because equipment depreciation significantly lowers the annual cost of a project. This subject is also the area of constant change because it changes as tax laws are revised. The amount of total capital investment in a project and the reduction in tax liability caused by depreciation or investment and energy tax credits or both, all have a major bearing on whether or not the project is economically feasible. However, the 1986 tax law drastically reduced many of these incentives. Competent tax accountants should be a part of the development team to ensure proper utilization of these considerations.

17.6.1 Straight Line Depreciation

Straight line depreciation is the simplest form of depreciation and has survived tax law changes for many decades and it is accepted by the 1987 tax law. The formula for straight line depreciation is:

$$\frac{cost - salvage}{life} = one\ unit\ of\ depreciation$$

Life can be expressed in years, months, units of production, operating hours, or miles. Under the 1987 tax law, salvage values are set to zero.

17.6.2 Sum-of-Years Digits Depreciation

This depreciation method is an accelerated depreciation schedule that recovers larger amounts of depreciation in the early life of the asset. The method of calculation is as follows:

1. Find the sum of the years' digits of the life of the equipment. Example: For a 10-year life, the sum of the digits 1 through 10 is equal to 55. The easiest way to make this calculation is:

$$\frac{life \times (life + 1)}{2}$$

in this case,

2. This sum is then divided into cost minus salvage to obtain one unit of depreciation, which is:

$$\frac{10 \times (10 + 1)}{2} = 55$$

Example: Equipment cost = \$60,000
Salvage value = \$ 5,000

then,

$$\frac{60,000 - 5,000}{55} = \$1,000 = one\ unit\ of\ depreciation$$

3. This unit of depreciation is then multiplied by the years of life in descending order, that is:

year 1 = 10 units = \$10,000
year 2 = 9 units = \$ 9,000
year 3 = 8 units = \$ 8,000
------ -- -- --------- -- ---------
year 10 = 1 unit = \$ 1,000.

The depreciation charge under this method performs like a negative arithmetic gradient where A_1 is \$10,000 and G is -\$1,000.

Although this method of depreciation is accepted accounting practice and may be used on equipment purchased before 1980, it is **no longer allowed under the 1987 tax law** and is only discussed to illustrate an application of the arithmetic gradient.

17.6.3 Declining Balance Depreciation

This method of depreciation is also an accelerated form and may obtain even larger depreciation in the early life than S-Y-D. With 200% declining balance depreciation, the annual rate is 200% times the straight line rate. As an example:

An asset with a 10-year life has a straight line rate of 1/10. Therefore, the 200% declining balance rate would be 2/10 or 20%.

This rate is **applied to the book value of the asset**, where book value equals cost minus accumulated depreciation. Any salvage value of the equipment was not considered except that the tax code provided that the equipment could not be depreciated below its salvage value. The \$60,000 piece of equipment in the example above would be evaluated using 200% declining balance as shown in Table 17.3.

Table 17.3 Declining Balance Depreciation Using 200% Declining Balance

Year	Book Value x Depreciation Rate ($)	Annual Depreciation ($)	Book Value End of Year ($)
1	60,000 x 0.20	12,000	48,000
2	48,000 x 0.20	9,600	38,400
3	38,400 x 0.20	7,680	30,720
-			
-			
-			

Although this method of depreciation provides the maximum write-off in the early life of the equipment, the annual depreciation charge rapidly decreases to the point that it would be beneficial to switch to straight line after the 6th year. It is interesting to note that the declining balance method of depreciation, whether it be 200%, 150% or 125%, takes the form of a negative geometric gradient and the book value for any year can be calculated using Equation (17.2). In this example,

$$Fv = \$60,000\ (1 - 0.20)^3$$
$$Fv = \$30,720 = \text{book value end of year 3},$$

to calculate the depreciation for year 6,

$$Fv = [60,000\ (1 - 0.20^5)]\ 0.20$$
$$Fv = \$3,932,16.$$

In the example above, the book value is calculated for the end of year 5, and multiplied by the depreciation rate, 0.20, to obtain the annual depreciation for year 6.

Once again, although this method of depreciation is an accepted accounting method, **declining balance depreciation was made obsolete with the 1980 tax law changes**. However, a modified version of this method was reinstated with the 1986 tax code revision and is discussed below.

17.6.4 Modified Accelerated Cost Recovery System (MACRS)

The Accelerated Cost Recovery System (ACRS) was introduced in 1981, but underwent major revision in 1986, effective with the 1987 tax year. In 1989, the Modified Accelerated Cost Recovery System (MACRS) was introduced, which was another major revision. MACRS depreciation is designed to provide rapid depreciation and to eliminate disputes over depreciation methods, useful life, and salvage value. The depreciation method and useful life are fixed by law, and the salvage value is treated as zero. MACRS depreciation rates depend on the recovery period for the property and whether the mid-year or mid-quarter convention applies. Property with class lives of 3, 5, 7, and 10 years may be depreciated at either the 200% or the 150% declining balance rate, with a switch to straight line. Property with class lives of 15 and 20 years must be depreciated using the 150% declining balance rate with a switch to straight line. In either case, the switch to straight line occurs when the straight line rate provides larger annual deductions. Table 17.4 lists the various classes of depreciable property under the Modified Accelerated Cost Recovery System.

Table 17.4 MACRS Class Lives

Property Class	All personal property other than real estate.
3-Year Property	Special handling devices used in the manufacturing of food and beverages.
	Special tools and devices used in the manufacture of rubber products, fabricated metal products, or motor vehicles, and finished plastic products.
	Property with a class life of 4 years or less.
5-Year Property	Automobiles, light-duty trucks (unloaded weight of less than 13,000 pounds).
	Semi-conductor manufacturing equipment.
	Aircraft owned by non-air transport companies
	Typewriters, copiers, duplicating equipment, heavy general purpose trucks, trailers, cargo containers, and trailer mounted containers.
	Computers.
	Computer-based telephone central office switching equipment, computer-related Peripheral equipment, and property used in research and experimentation.
	Equipment qualifying as a small power production facility within the meaning of Section 3(17)(C) of the Federal Power Act (16 U.S.C. 796 (17)(C)), as in effect on 9-1-86.
	Petroleum drilling equipment.
	Property with a class life of more than 4 and less than 10 years.

Table 17.4 MACRS Class Lives (continued)

Property Class	All personal property other than real estate.
	All property not assigned by law to another class.
	Any railroad track.
7-Year Property	Office furniture, equipment, and fixtures. Cellular phones, fax machines, refrigerators, dishwashers. Machines used to produce jewelry, toys, musical instruments, and sporting goods.
	Single-purpose agricultural or horticultural structures placed in service in 1987 or 1988.
	Property with a class life of 10 years or more, but less than 16 years.
	Equipment used in the refining of petroleum, the manufacture of tobacco products and certain food products.
	Railroad cars.
10-Year Property	Water transportation equipment and vessels.
	Single-purpose agricultural or horticultural structures place in service after 1988.
	Property with a class life of 16 years or more, but less than 20 years.
	Land improvements such as fences, shrubbery, roads, and bridges.
	Any municipal waste water treatment plant.
15-Year Property	Telephone distribution plants and equipment used for 2-way exchange of voice and data communications.
	Property with a class life of 20 years or more, but less than 25 years.
	Farm buildings.
20-Year Property	Municipal sewers.
	Property with a class life of 25 years or more.
27.5-Year Property	Residential rental property (excluding hotels and motels) placed in service after December 31, 1986.
31.5 Year Property	Non-residential real property placed in service after December 31, 1986, but before May 13, 1993.
39-Year Property	Non-residential property placed in service after May 12, 1993.

The mid-year convention treats all property acquired during the year as though it were acquired in mid-year and only half of the first year depreciation is allowed. Similarly, in the year after the last class life year, the remaining depreciation is written off. If property is sold, only half of the full depreciation for the year of sale is allowed. Therefore, if the $60,000 piece of equipment is depreciated under MACRS, the depreciation schedule would be as shown in Table 17.5

Table 17.5 Depreciation Schedule

Year	Book Value x Depreciation Rate	Annual Depreciation	Book Value End of Year
Year	($)	($)	($)
1	60,000 x 0.10	6,000	54,000
2	54,000 x 0.20	10,800	43,200
3	43,200 x 0.20	8,640	34,560
--			
--			
--			

Under this system of depreciation, the user would switch to straight line in year 7. Table 17.6 provides values for the various classes of equipment and these values switch to straight line automatically in the year that straight line will provide a larger annual depreciation. In year 7, 8, 9, and 10, the depreciation on the $60,000 piece of equipment would be $3,900/year. However, in year 11, which is considered the year of disposal (for tax purposes), the mid-year convention would allow only $1,965 of depreciation.

Table 17.6 provides depreciation percentage values for Modified Accelerated Cost Recovery System using the mid-year convention.

Figure 17.5 illustrates an asset costing $50,000 with zero salvage value and a 10-year life depreciated by all of the above methods.

During years 7 through 10, MACRS and 200% declining balance are almost identical. The property is fully depreciated at the end of year 10 using 200% declining balance; but, using MACRS, the mid-year convention applies in year 11.

The mid-quarter convention applies when more than 40% of all property placed in service during the year is acquired during the last three months of the year. Under the mid-quarter convention, property placed in service in any quarter is treated as though it were acquired in mid-quarter and only half of the depreciation for that quarter is allowed. That is, property acquired in the first quarter would be allowed 3.5 quarters of depreciation in the first year. Property acquired in the third quarter would be allowed 1.5

Table 17.6 MACRS Depreciation Values Mid-Year Convention

Recovery Year	3-Year Class	5-Year Class	7-Year Class	10-Year Class	15-Year Class	20-Year Class
1	33.33%	20.00%	14.29%	10.00%	5.00%	3.75%
2	44.44	32.00	24.49	18.00	9.50	7.22
3	**14.81**	19.20	17.49	14.40	8.55	6.68
4	7.41	**11.52**	12.49	11.52	7.70	6.18
5		11.52	**8.92**	9.22	6.93	5.71
6		5.76	8.92	7.37	6.23	5.28
7			8.92	**6.55**	**5.90**	4.89
8			4.46	6.55	5.90	4.52
9				6.55	5.90	**4.46**
10				6.55	5.90	4.46
11				3.28	5.90	4.46
12					5.90	4.46
13					5.90	4.46
14					5.90	4.46
15					5.90	4.46
16					2.95	4.46
17						4.46
18						4.46
19						4.46
20						4.46
21						2.23

Notes: The table values are to be multiplied by the cost basis of the equipment.

3-year class through 10-year class is depreciated using 200% declining rate, converting to straight line in the year underlined.

15- and 20-year class property is depreciated using the 150% declining rate converting to straight line in the year underlined.

The half-year convention treats all classes as though they were placed in service in mid-year, allowing 0.5 y of depreciation in year 1 and 0.5 y of depreciation when the property is disposed of, removed from service, or in the last recovery year.

quarters of depreciation in the first year. Similarly, in the year after the last class life year, the remaining quarters of depreciation are written off.

Mid-quarter values to multiply by the cost basis of the property may be calculated as follows:

Let remaining value (rv) = (1 - accumulated depreciation)
Let remaining periods (rp) = (1 - accumulated periods)

then:

v = the greater of [(r/1)(rv)(np) or (rv/rp)(np)].

where,

v = the percentage value to multiple by the cost basis
r = either 200% or 150% declining balance rate
l = Property Class Life in terms of years or quarters
np = the number of periods allowed in the current year
rv = the remaining property value
rp = the remaining periods.

Example: 17.20: MACRS Mid-Quarter Convention.

Property with a class life of 5 years, purchased during the second quarter, is to be depreciated by the mid-quarter convention, using the 200% Declining Balance Rate. Values to be multipled by the cost basis of the equipment could be generated using the above formulas as follows:

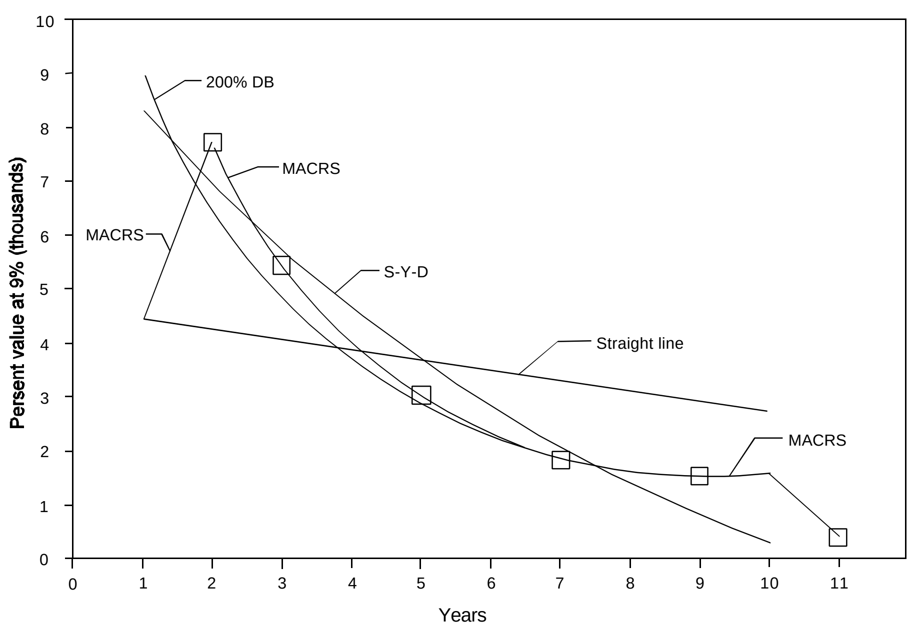


Figure 17.5 Depreciation methods.

Year	Value (m)	Declining Balance (r/l)(rv)(np)	Straight Line (rv/rp)(np)	Periods Allowed (np)	Remaining Value (rv)	Remaining Periods (rp)
1	25.00%	**25.00%**	12.50%	2.5	100.00%	20.0
2	30.00	**30.00**	17.14	4	75.00	17.5
3	18.00	**18.00**	13.33	4	45.00	13.5
4	11.37	10.80	**11.37**	4	27.00	9.5
5	11.37	6.25	**11.37**	4	15.63	5.5
6	4.26	0.64	**4.26**	1.5	4.26	1.5

The underlined values are multiplied by the cost basis of the property to determine the amount of depreciation allowed in each respective year.

The Internal Revenue Code also provides for first-year expensing of equipment placed in service in 1997. The first-year expensing is technically called the "Section 179 Deduction." The deduction is limited to $18,000 in 1997 and will increase in 1998. The deduction phases out when equipment costing over $200,000 is placed in service in any one year. The deduction is not allowed for buildings, structural components of buildings, and furnishings of residential rental property. First-year expensing must be subtracted from the cost basis of the equipment for depreciation purposes.

The Internal Revenue Code and the annual changes that occur are beyond the scope of this text. Users must maintain current knowledge of the tax law in order to be certain they are in compliance with existing guidelines.

17.7 LIFE CYCLE COST ANALYSIS

Life cycle costing (LCC) evaluates all the costs associated with acquisition, construction and operation of a project. LCC is designed to minimize costs of major projects, not only in consideration of acquisition and construction, but especially in the reduction of operation and maintenance costs during the project life. LCC is the calculation of all annual costs and revenues over the life of the project. These values are totaled by year and discounted back to time zero at some interest rate to arrive at a net present value. This process is repeated for each alternative. The alternatives are then compared, based on net present value or equivalent annual cost.

When performing this analysis, it is important to do sensitivity analysis. Sensitivity analysis consists of changing parameters or variables within the project to determine their effect on the feasibility of the project. Sensitivity analysis is accomplished by substituting one type of construction for another or evaluating various pieces of equipment with different operating costs, evaluating what effect a change in the economic inflation rate would have on the project, and considering various financing scenarios to observe their effect on the outcome.

Example 17.21: Table 17.7 provides the input data for evaluating three heating systems. As a very simple example of LCC, consider a 15-year LCC for three alternatives (A, B, and C as shown in Tables 17.8, 17.9, 17.10, 17.11, 17.12, and 17.13) to provide heat for a 30,000 ft^2 office building.

Table 17.7 Heating System Cost Alternatives

Heating	Electric Resistance	Heat Pump	Geothermal
Capital cost ($)	158,400	180,000	233,100
Life (y)	12	12	12
Salvage value ($)	-0-	-0-	-0-
Annual electricity requirement (kWh)	263,680	131,840	22,620
Cost ($/kWh)	$ 0.05	$ 0.05	$ 0.05
Annual maintenance ($)	1,584	2,650	2,331
Annual insurance ($)	554	630	816
Annual property tax ($)	238	270	350
Compressor replacement end of year 10 ($)	-0-	750	-0-

Tables 17.8, 17.9, and 17.10 provide a 15-year LCC on each system by year and evaluate these costs at 8% compounded annually. Electrical power costs are assumed to be $0.05/kWh. Notice each cost is entered in the year it occurs. Insurance premiums are paid annually and are in the form of an annuity due. Based on the present value at an 8% rate of interest over the 15-year life cycle, the heat pump has the lowest cost, and based on the criteria of the lowest present value, the heat pump would be the best selection. However, when the cost of electricity is increased to $0.07/kWh, presented in Tables 17.11 and 17.12, the geothermal system becomes less expensive, having a lower net present value, and lower annual equivalent costs.

Although the annual equivalent cost accurately indicates which alternative is the best choice, it has absolutely nothing to do with actual expenditures per year. Although the Heat Pump costs $6,592/y to operate and has higher maintenance costs than the Geothermal system, at $0.05/kWh it provides the lowest annual cost. The reason for this is the lower initial cost of the Heat Pump.

Another approach to LCC analysis would be to examine the difference in costs between System B and System C. Table 17.13 illustrates this approach. Note that the cost of electricity has been changed to $0.07/kWh. Although $53,100 additional is spent by purchasing System C over System B, the net present value of the operating costs over a 15-year period at 8%/y is $13,019 more for System B than for System C. Because the savings in operation and maintenance costs exceed the value of the initial investment, System C would be chosen over System B.

Table 17.8 Life Cycle Cost of Heating System (A)

Electric Resistance Heat

Capital Cost	$158,400
Interest Rate	8%
Electric Power Cost	$0.05/kWh

Year	Electric Power Cost ($)	Ins. Cost ($)	Prop. Tax Cost ($)	Annual Maint. Cost ($)	Total Cost ($)	Present Value ($)
0	-0-	554	-0-	-0-	158,954	158,954
1	13,184	554	238	1,584	15,560	14,407
2	13,184	554	238	1,584	15,560	13,340
3	13,184	554	238	1,584	15,560	12,352
4	13,184	554	238	1,584	15,560	11,437
5	13,184	554	238	1,584	15,560	10,590
6	13,184	554	238	1,584	15,560	9,805
7	13,184	554	238	1,584	15,560	9,079
8	13,184	554	238	1,584	15,560	8,407
9	13,184	554	238	1,584	15,560	7,784
10	13,184	554	238	1,584	15,560	7,207
11	13,184	554	238	1,584	15,560	6,673
12	13,184	554	238	1,584	15,560	6,179
13	13,184	554	238	1,584	15,560	5,721
14	13,184	554	238	1,584	15,560	5,298
15	13,184	-0-	238	1,584	15,006	4,730

Total Cost	$ 391,800
Net Present Value	$ 291,965
Annual Equivalent Cost	$ 34,110

Table 17.9 Life Cycle Cost of Heating System (B)

Air-to-Air Heat Pump

Capital Cost	$180,000
Interest Rate	8%
Electric Power Cost	$0.05/kWh

Year	Electric Power Cost ($)	Ins. Cost ($)	Prop. Tax Cost ($)	Annual Maint. Cost ($)	Total Cost ($)	Present Value ($)
0	-0-	630	-0-	-0-	180,630	180,630
1	6,592	630	270	2,650	10,142	9,391
2	6,592	630	270	2,650	10,142	8,695
3	6,592	630	270	2,650	10.142	8,051
4	6,592	630	270	2,650	10,142	7,455
5	6,592	630	279	2,650	10,142	6,902
6	6,592	630	270	2,650	10,142	6,391
7	6,592	630	270	2,650	10,142	5,918
8	6,592	630	270	2,650	10,142	5,479
9	6,592	630	270	2,650	10,142	5,074
10	6,592	630	270	3,400[a]	10,142	5,045
11	6,592	630	270	2,650	10,142	4,350
12	6,592	630	270	2,650	10,142	4,028
13	6,592	630	270	2,650	10,142	3,729
14	6,592	630	270	2,650	10,142	3,453
15	6,592	-0-	270	2,650	9,512	2,999

Total Cost	$ 332,880
Net Present Value	$ 267,589
Annual Equivalent Cost	$ 31,262

a. Indicates compressor replacement.

Table 17.10 Life Cycle Cost of Heating System (C)

Geothermal Heating System

Capital Cost	$233,100
Interest Rate	8%
Electric Power Cost	$0.05/kWh

Year	Electric Power Cost ($)	Ins. Cost ($)	Prop. Tax Cost ($)	Annual Maint. Cost ($)	Total Cost ($)	Present Value ($)
0	-0-	816	-0-	-0-	233,916	233,916
1	1,131	816	350	2,331	4,628	4,285
2	1,131	816	350	2,331	4,628	3,967
3	1,131	816	350	2,331	4,628	3,673
4	1,131	816	350	2,331	4,628	3,401
5	1,131	816	350	2,331	4,628	3,149
6	1,131	816	350	2,331	4,628	2,916
7	1,131	816	350	2,331	4,628	2,700
8	1,131	816	350	2,331	4,628	2,500
9	1,131	816	350	2,331	4,628	2,315
10	1,131	816	350	2,331	4,628	2,143
11	1,131	816	350	2,331	4,628	1,985
12	1,131	816	350	2,331	4,628	1,838
13	1,131	816	350	2,331	4,628	1,702
14	1,131	816	350	2,331	4,628	1,575
15	1,131	816	350	2,331	3,812	1,202

Total Cost	$ 302,513
Net Present Value	$ 273,268
Annual Equivalent Cost	$ 31,962

Table 17.11 Life Cycle Cost of Heating System (B)

Air-to-Air Heat Pump

Capital Cost	$180,000
Interest Rate	8%
Electric Power Cost	$0.07/kWh

Year	Electric Power Cost ($)	Ins. Cost ($)	Prop. Tax Cost ($)	Annual Maint. Cost ($)	Total Cost ($)	Present Value ($)
0	-0-	630	-0-	-0-	180,630	180,630
1	9,229	630	270	2,650	12,779	11,832
2	9.229	630	270	2,650	12,779	10,956
3	9,229	630	270	2,650	12,779	10,144
4	9,229	630	270	2,650	12,779	9,393
5	9,229	630	270	2,650	12,779	8,697
6	9,229	630	270	2,650	12,779	8,053
7	9,229	630	270	2,650	12,779	7,456
8	9,229	630	270	2,650	12,779	6,904
9	9,229	630	270	2,650	12,779	6,393
10	9,229	630	270	3,400[a]	12,779	6,266
11	9,229	630	270	2,650	12,779	5,481
12	9,229	630	270	2,650	12,779	5,075
13	9,229	630	270	2,650	12,779	4,699
14	9,229	630	270	2,650	12,779	4,351
15	9,229	630	270	2,650	12,149	3,830

Total Cost	$ 372,432
Net Present Value	$ 290,159
Annual Equivalent Cost	$ 33,899

a. Indicates compressor replacement.

Table 17.12 Life Cycle Cost of Heating System (C)

Geothermal Heating System

Capital Cost	$233,100
Interest Rate	8%
Electric Power Cost	$0.07/kWh

Year	Electric Power Cost ($)	Ins. Cost ($)	Prop. Tax Cost ($)	Annual Maint. Cost ($)	Total Cost ($)	Present Value ($)
0	-0-	816	-0-	-0-	233,916	233,916
1	1,583	816	350	2,331	5,080	4,704
2	1,583	816	350	2,331	5,080	4,355
3	1,583	816	350	2,331	5,080	4,033
4	1,583	816	350	2,331	5,080	3,734
5	1,583	816	350	2,331	5,080	3,457
6	1,583	816	350	2,331	5,080	3,201
7	1,583	816	350	2,331	5,080	2,964
8	1,583	816	350	2,331	5,080	2,745
9	1,583	816	350	2,331	5,080	2,541
10	1,583	816	350	2,331	5,080	2,353
11	1,583	816	350	2,331	5,080	2,179
12	1,583	816	350	2,331	5,080	2,017
13	1,583	816	350	2,331	5,080	1,868
14	1,583	816	350	2,331	5,080	1,730
15	1,583	816	350	2,331	4,264	1,344

Total Cost	$ 309,299
Net Present Value	$ 277,140
Annual Equivalent Cost	$ 32,378

Table 17.13 Incremental Difference of System (C-B)

Geothermal minus Heat Pump

Incremental Cost	$ 53,100
Interest Rate	8%
Electric Power Cost	$0.07/kWh

Year	Electric Power Cost ($)	Ins. Cost ($)	Prop. Tax Cost ($)	Annual Maint. Cost ($)	Total Cost ($)	Present Value ($)
0	-0-	816	-0-	-0-	-53,286	-53,286
1	7,645	186	80	319	7,699	7,129
2	7,645	186	80	319	7,699	6,601
3	7,645	186	80	319	7,699	6,112
4	7,645	186	80	319	7,699	5,659
5	7,645	186	80	319	7,699	5,240
6	7,645	186	80	319	7,699	4,852
7	7,645	186	80	319	7,699	4,492
8	7,645	186	80	319	7,699	4,159
9	7,645	186	80	319	7,699	3,851
10	7,645	186	80	1,069	8,449	3,913
11	7,645	186	80	319	7,699	3,302
12	7,645	186	80	319	7,699	3,057
13	7,645	186	80	319	7,699	2,831
14	7,645	186	80	319	7,699	2,621
15	7,645	-0-	80	319	7,699	2,486

Total Cost	$ 63,134
Net Present Value	$ 13,019
Annual Equivalent Cost	$ 1,521
Incremental Rate of Return on System ©	11.7986%

Another method of evaluation involves calculating the ROR of the savings versus the costs. This method is also illustrated in Table 17.13. By iteration (trial and error), a rate of 11.7986% causes the net present value of the savings to be exactly equal to the net present value of the costs. Therefore, the ROR realized by selecting System C over System B is approximately 12% annually. Often in this type of analysis, the actual annual costs are not an equal stream of payments, and may frequently swing from positive to negative. Such detailed analysis on uneven cash flows can only be performed efficiently with a computer program.

Tables 17.8 through 17.13 do not contain any form of a gradient. If any of the costs are allowed to inflate, the problem would become much more complex. Furthermore, all costs would not inflate at the same rate. The price of a replacement system 15 years from today would inflate at one rate, electricity at a different rate, and insurance would probably be a different rate. However, with the use of a microcomputer it would be possible to establish different inflation rates for all cost data and perform the same analysis as shown in Tables 17.8 through 17.13.

These three alternatives are extremely sensitive to the cost of electrical power.

17.8 RELCOST PROGRAM

LCC on a major project requires thousands of calculations involving every formula presented thus far in this chapter with the possible exception of arithmetic gradients. However, with the advent of the microcomputer and software designed to do **preliminary** LCC analysis, the cost analyst can provide a relatively good guess as to the economic feasibility of a project.

The Renewable Energy Life Cycle COST program, RELCOST, was designed for just this purpose. Any LCC analysis of a project requires an extensive accompanying report in order to explain in detail how values were calculated. RELCOST provides an output, at the option of the user, that will print all of the input data, the energy forecast, the depreciation schedule, and all of the output data in tabular form, minimizing the need for an accompanying report. RELCOST also provides an output file accessible to Harvard Graphics.

The program was developed through the Washington State Energy Office using funds provided by the state energy offices of Washington and Oregon, and from private sources. The 150 page user manual is supplied on a floppy disk. To obtain copies of this program, contact the Washington State University Energy Program, PO Box 43165, Olympia, WA 98504-3165, telephone: 360-956-2016.

The RELCOST Program, Release 2.1, can accommodate inflation rates for 13 different energies, and it is possible to change the inflation rate for each of these energies every year throughout an energy forecast of up to 30 years. In addition, RELCOST will accommodate up to five different classes of equipment with varying useful lives and with different depreciation schedules. Inflation rates regarding equipment replacement can vary for each equipment class.

RELCOST allows up to 5 years for construction and acquisition. During this construction period, it allows any combination of project financing to include bonds, bank loans, and equity. It also allows capital investment during the construction period for project funds that are in excess of project costs for any particular year. The life of bank loans and bond issues can vary year by year, and interest rates may differ for bonds, bank loans, and investments for each year of construction. The program permits 12 different depreciation schedules for equipment, including the half-year convention, and these rates may be altered by the user. The program makes provisions for percentage depletion allowances, investment tax credits, and energy tax credits. It can accept up to three annual revenue streams, inflating at some user-defined rate per year over the life of the project. It also allows for a growth rate in the number of customers on-line in any given year.

RELCOST provides for four resident energy forecasts, up to 30 years in length. Once all the input data has been collected, and energy forecasts have been established, the program requires less than 30 minutes to enter the data and do the calculations for a major project.

A complete run on a hypothetical heating district for a taxable entity is provided in Tables 17.14 thru 17.22. The same project for a non-taxable entity appears in Tables 17.23 thru 17.29.

The typical life-cycle cost input data supplied considers the difference between an existing system and a proposed system; the same type of analysis that was performed using System B and System C in Table 17.13. That is, the program considers the initial cost of the proposed system and evaluates the annual savings provided by the proposed system versus the capital investment required for the proposed system to determine whether or not the system is economically feasible. It is not necessary to have a present system. LCC analysis could be accomplished on a proposed system only. However, in order to perform this analysis, the program would have to consider the initial cost of the proposed system versus the revenues generated by the proposed system.

Table 17.14 illustrates input screen one, two, and three of the RELCOST Program. Screen one consists of the report title, location, date, and gives the user the option of choosing one of four forecast files or creating an energy forecast for this report only. These forecast files must be created, maintained, and updated by the user.

Screen two begins with capital investment data. Data entered on this sheet includes the beginning year of investment, construction period, economic inflation rate, discount rate or cost of capital selected for the project and all financing data in the form of owner's equity, bonds, and bank loans used to finance the project.

Input screen three, which appears in the lower third of Table 17.14, includes all present costs of energy for the existing and the proposed system. The first five energies: natural gas, fuel oil, propane, coal, and electricity, are the conventional fuels supplied for every project. The next five energies included under the heading Other Energy" are user selectable. The titles of these fuels are obtained from the energy forecast. If no titles are supplied by the user, then they are listed as "Other 1" through "Other 5." The user may elect to use any one or all of these fuels in the project under study.

The last two items on screen three: property tax and insurance, and operation and maintenance, have their own independent inflation rates that are not in addition to the economic inflation rate. However, if the user wanted these rates to inflate differently year by year, they could be entered on an energy forecast as an energy, properly titled, and these two items could be omitted at the bottom of screen three.

Screen four Table 17.15 presents energy sales for both the present and proposed systems. This screen also allows percentages depletion allowance for energy sales under the proposed system. The amount of the percentage depletion allowance is determined by the user on the depreciation schedule under depletion, and if necessary, could change year by year, or as the tax code changes. The 1986 tax code changed percentage depletion allowance from 20 to 22%, effective in 1987. The user must consult the current tax code to determine those energies eligible for depletion allowance and the percent of depletion allowance. The program does not provide cost depletion, although the user could calculate such depletion in the depreciation section. Depletion allowances are only available for taxable entities.

Screen five on Table 17.15 presents five classes of equipment. This screen allows the user to group all equipment associated with a project into five different classes depending upon economic life (the actual usable life of the equipment), taxable life, actual expected salvage value, taxable salvage value, and the method of depreciation to be applied against that class of equipment. This portion of the input section also allows inflation rates for subsequent purchases of equipment and inflation rates for actual salvage value to determine equipment replacement costs throughout the life cycle of the project. The user should be aware that equipment is automatically replaced at the end of its economic life without any regard for the life of the project. In other words, if a class of equipment had a five-year economic life, the project had a three-year construction period and a life-cycle cost analysis was done for a 20-year period, that class of equipment would auto-matically be replaced at the beginning of year 9, 14, and 19.

There are 12 columns of depreciation schedules available in the depreciation file. The data in these columns can be modified by the user and hopefully will be adequate for future changes to the tax law. Columns one through six, supplied with the program, provide ACRS depreciation schedules for 3-, 5-, 7-, 10-, 15-, and 20-year lives, using the half-year convention. The user may also select straight line depreciation. Selecting this option will cause the program to calculate the annual depreciation charge for that class of equipment based on its cost basis minus any salvage that may be allowed, for either a full-year or half-year convention, depending on the option selected by the user. All depreciation schedules are modifiable except for straight line.

Column zero supplies multipliers for percentage depletion allowances.

The last screen on Table 17.15 is screen six, which requires income tax and investment tax credit input data. This screen allows the user to select the marginal federal and state tax rates, the percentage of investment tax credit or energy tax credit available or both, and the amount of the capital investment eligible for these tax credits.

Table 17.16 is an example of a typical energy forecast. The inflation rates by year and by type of energy appear in the columns as decimal amounts and represent an inflation rate above or below the economic inflation rate for each fuel. In other words, if the inflation rate in the natural gas column was 0.02 and the economic inflation rate was 0.06, this would indicate that natural gas was expected to inflate at 8% in year one. Inflation rates entered in the forecast may be positive, negative or zero. A zero indicates that the energy is expected to inflate at the same rate as the economic inflation rate.

Table 17.17 illustrates the depreciation and depletion allowance schedule. All values in these tables may be modified by the user. However, the energy forecast is limited to 30 years. If the project life goes beyond 30 years, the inflation rate for year 30 is chosen as the inflation rate for the last 10 years of the project.

Tables 17.18 through 17.22 provide the output data for the project. Total project cost, equity financing and debt financing are all stated in terms of net present value; that is, all monies flowing into the project during the construction period are brought back to time zero at the discount rate.

The benefit-cost ratio evaluates the net present value of the annual savings or revenues associated with the proposed system or both, and divides this number by the net present value of the total project cost.

The net present value calculates the present value of the total project costs and subtracts the present value of the annual savings or revenues associated with the project or both, to arrive at a net present value for the total project, or a net present value for the life cycle of the project.

A RELCOST user developed a scenario on a renewable energy project with equipment that required replacement every 6 months and he wished to evaluate this with equipment replaced every 18 months. In order to accomplish this, all that was necessary was to state replacement costs, all operating costs, all inflation rates, the economic inflation rate, capital investment costs and financing in terms of 1 month rather than 1 year, respectively, and complete a 36-month life cycle cost analysis. There are numerous ways to modify the input data in order to provide for unusual circumstances.

17.9 CAVEATS

As was stated in the introduction, LCC has several major drawbacks. One of these is that increasing or decreasing costs over the life of the project must be estimated based on some forecast, and forecasts have proven to be highly variable and frequently inaccurate. Another problem with LCC is that some life span must be selected over which to evaluate the project, and many projects, especially renewable energy projects, are expected to have an unlimited life; they are expected to live forever. **The longer the life cycle, the more inaccurate annual costs become because of the inability to forecast accurately**.

Comprehensive LCC should be performed by qualified persons who have a thorough knowledge of the subject, including expertise in the current tax law.

17.10 RECOMMENDATIONS

Based on the experience gained by the author in completing over 150 economic analysis for renewable energy projects, the following recommendations are offered.

17.10.1 Take the Time to Understand the Basics

Preliminary LCC requires that the user understand certain basic concepts regarding interest, taxes, the time value of money, and economic decision techniques for even the simplest analysis. If you have no experience in these areas, carefully read this chapter and proceed with extreme caution. If certain concepts remain unclear, consult the references appearing at the end of this chapter.

17.10.2 Use Ultraconservative Forecasts

During the petroleum crisis in the mid 1970s, forecasters were predicting inflation rates for fossil fuels ranging from 5 to 15% above the economic inflation rate. Many of these energy forecasts were 20 years in length. Using hindsight, it turns out that within the last 10 years petroleum prices have actually deflated when compared to the consumer price index. Any forecast that projects inflation rates higher than those that occur provides a favorable LCC picture, gives a green light to the project, causes thousands and even millions of dollars to be spent in acquisition and construction, only to find that when the project goes into production, competing fuels have not inflated as projected and the project either operates at a continual loss or is abandoned.

17.10.3 Keep Project Life as Short as Possible

It is dangerous to try to do even a 20-year study on a major project. It is absolutely ridiculous to go beyond 20 years. And yet, many projects have required 50-year LCC analysis. The successful projects are those that use extremely low inflation rates and have a very short payback (4 years or less).

LCC analysis on a large project is a very complex procedure. Many projects require equipment with widely varying useful lives. Pipe lines may last 25 years. Electric motors, turbines, or pumps may have useful lives of 5 to 10 years. Replacement costs must be forecast, major repair costs have to be incorporated, and competing alternatives should be as accurately evaluated as possible. If the project is owned by a private entity, depreciation schedules, depletion allowances, investment tax credits, energy tax credits, and in some cases, intangible drilling costs enter the picture.

It is not possible to predict future changes in the tax law. In the early 1980s, many renewable energy projects enjoyed as much as a 55% tax credit. Later (5 years), many of these tax credits were removed. Therefore, any equipment replaced could not provide the reduced tax liability that was forecast at the beginning of the project.

17.10.4 Minimize Total Project Cost

As observed in the LCC analysis of the three heating systems, initial cost probably has the greatest impact on the feasibility of a project and the developer should make every effort to minimize the project cost without sacrificing quality and reliability. For example, in the development of geothermal projects, drilling costs, and pipeline costs are two of the major components. In an attempt to minimize drilling costs, quality, performance and longevity of the production well may be sacrificed. It may be advisable to move the user on-site to minimize the length of transmission lines without sacrificing any other aspect of the project. The end result is a reduction of pipeline maintenance and pumping costs.

17.10.5 Carefully Evaluate Financing Options

The method of financing, the interest rate, and the annual debt service can have a major impact on renewable energy projects. These projects are capital intensive and experience very high costs during the construction period. In addition, many projects come on-line gradually, which causes very low revenues in the early years of the project life. In such cases, it may be advisable to seek long-term bond financing. Such financing requires only interest payments during the early years of the project, and a high balloon payment when the bond matures. As the project reaches full capacity in the later years, revenues may be able to accommodate such a balloon payment. Such financing can be especially helpful to municipalities experiencing low or negative cash flows during the beginning years of the project.

17.10.6 Avoid Distorting the Rate of Return

This author discourages using financial leverage to make a project appear feasible. Financial leverage is the practice of financing large sums of money at extremely low interest rates in order to reduce the equity invested in the project and increase the ROR. For example, a project costing $100,000 returns $15,000 in revenues every year for 10 years. The ROR on this project would be about 8%. In an effort to make the project appear more attractive, the developer borrows $90,000 at 6% interest, which requires an annual payment of $12,228. Subtracting the debt service from the income provides a net annual income of $2,772. Now, the project is evaluated as a $10,000 investment providing $2,772 of income per year for 10 years and the ROR jumps to nearly 25%. Once project feasibility is determined, it is up to the developer to use any accepted method to increase the project's ROR; but, such practices should be avoided in determining whether or not the project is economically feasible.

17.10.7 Be Aware of the Limitations of Payback

Simple payback compares the values in the cumulative cash flow column (Table 17.22 column 22) with the equity invested in the project to determine when these two values are equal. This practice makes no distinction between present and future values. Note that the simple payback cash flows are not evaluated at any interest rate and that a negative cash flow occurring subsequent to the payback period has no effect. If the cumulative cash flows indicate that the equity in a project is recovered in year 12, and year 13 has a balloon payment on a bond that causes the cash flow to turn negative, this may cause the simple payback to be in error.

The discounted payback compares the equity in the project with the cumulative discounted cash flow (Table 17.22 column 24) and indicates a discounted payback when these two are equal. Once again, this method does not evaluate subsequent negative cash flows that may occur.

Nonetheless, it is a more accurate measure of payback because the accumulated cash flows are discounted at the discount rate stated in the input section.

The reason the simple and discounted payback periods are evaluated in this manner is because it is the accepted calculation method in the industry. Because these values may be in error, the program RELCOST also evaluates cash flows beyond these payback periods and prints a warning in the output section if negative cash flows occur beyond the payback period.

The LCC analysis is a very powerful tool. It can be used to rank projects in the order of feasibility and to determine which projects are most likely to be successful. At best however, it is only an educated guess and the values calculated should be treated only as rough estimates.

NOTE: See Glossary and References List at end of Chapter.

Table 17.14

LIFE CYCLE COST ANALYSIS
for
The Horsefly Heating District
Klamath Falls, Oregon
April 28, 1987

Energy forecast used for this report: 1986 Sample Energy Forecast.* This report is for a taxable entity. Dollar values rounded to the nearest: 1

Capital investment data:

Year Of Initial Capital Investment...............................(example: 1988)...........:	1988
Year Project Will Be In Production............................(example: 1990)............:	1992
Project Life In Years.....................(enter a whole number from 1 to 40)...........:	20
Economic Inflation Rate......................(enter as a whole number e.g. 7)...........:	4%
Discount Rate................................(enter as a whole number 3.g. 12)..............:	10%

Financing:

Beginning of Year	1992	1991	1990	1989	1988
Pr Jt Cost	....	500,000	300,000	700,000	500,000
Equity	....	60,000	....	....	250,000
Bank Loan	....	400,000	....	....	800,000
Life	....	10	....	....	15
(APR)	...........%	10%	...........%	...........%	9%
Bond Issue	....	....	....	....	1,000,000
Life	....	....	....	....	18
(APR)	...........%	...........%	...........%	...........%	6%
Invested	....	1,157	241,366	642,765	1,390,752
(APR)	...........%	8.5%	10%	9%	8%

Investment EOY 1992: 1,255 Total Project Cost (NPV): 1,759,954

Year Zero Annual Costs:	Present System	Proposed System
Conventional Energy:		
Natural Gas..	50,000	400
Fuel Oil..	....	....
Propane...	....	....
Coal...	....	....
Electricity..	150,000	100,000
Other Energy:		
Geothermal..	....	10,000
Solar..	....	....
Nuclear..	....	....
Biomass...	....	....
Waste Heat..	....	....
Property Tax and Insurance..	2,500	3,000
Inflation Rate...		1.5%
Operation and Maintenance..	15,000	25,000
Inflation Rate...		4%

Table 17.15

Year Zero Annual Costs:	Present System	Proposed System
Sales of Energy:		
Steam	....	....
Hot Water	....	220,000
Tipping Fees	....	....

Percentage Depletion Allowance (for proposed system only):
(See column 13 of Depreciation Schedule for rate.)

Indicate sales eligible	Steam:	No
for percentage depletion	Hot Water:	Yes
allowance: (Y/N)	Tipping Fees:	No

Taxable income limitations for depletion allowance: 50%
(Based on income resulting from sales eligible for depletion.)

Equipment Purchases:	Class 1	Class 2	Class 3	Class 4	Class 5
Initial Cost:	500,000	250,000	120,000	90,000	....
Yr. Purchased:	1988	1989	1990	1990	....
Inflation/yr:	4%	..........%	5%	..........%	6%
Actual Slvg.:	50,000	....	20,000	....	....
Economic Life:	10	25	18	20	....
Taxable Slvg.:	1,000	....	....	....	....
Taxable Life:	5	10	7	12	....
Dpren. Method:	STLN	ACRS	ACRS	STLN	STLN
Half Yr. Rule:	No	No	No	Yes	No
Fed. Tax Credits:					
Business: (Y/N)	Yes	No	Yes	No	No
Energy: (Y/N)	No	No	Yes	No	No
Adjust Equipment Cost Basis By:	5%	..........%	12.5%	..........%	..........%

Income Tax and Investment Tax Credit Information:	Federal	State
Marginal Tax Rate	40%	10%
Business Investment Tax Credit:		
Amount Eligible	500,000	100
Rate	10%	10%
Energy Tax Credit:		
Amount Eligible	200,000	100
Rate	15%	15%

Table 17.16

Sample Energy Forecast*
Forecast Energy Inflation Rates

1986

Yr	1 Natu	2 Fuel	3 Prop	4 Coal	5 Elec	6 Geot (1)	7 Sola (2)	8 Nucl (3)	9 Biom (4)	10 Wast (5)	11 Stea (1)	12 Hot (2)	13 Tipp (3)
86	0.020	0.019	0.019	0.020	0.009	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
87	0.020	0.019	0.019	0.020	0.009	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
88	0.020	0.019	0.019	0.020	0.009	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
89	0.020	0.019	0.019	0.020	0.009	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
90	0.022	0.019	0.019	0.020	0.009	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
91	0.022	0.019	0.019	0.020	0.009	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
92	0.022	0.019	0.019	0.020	0.009	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
93	0.022	0.019	0.019	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
94	0.022	0.019	0.019	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
95	0.030	0.019	0.019	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
96	0.030	0.019	0.019	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
97	0.030	0.019	0.019	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
98	0.030	0.019	0.019	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
99	0.030	0.019	0.019	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
00	0.032	0.018	0.018	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
01	0.032	0.018	0.018	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
02	0.032	0.018	0.018	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
03	0.032	0.018	0.018	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
04	0.032	0.018	0.018	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
05	0.032	0.018	0.018	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023

Table 17.17

Equipment Depreciation Schedule

		1	2	3	4	5	6	7	8	9	10	11	12
Method		ACRS	ACRS	ACRS	ACRS	ACRS	ACRS	ACRS	ACRS				
Life		3-Yr	5-Yr	7-Yr	10-Yr	15-Yr	20-Yr	3-Yr	5-Yr				
	DPLTN												
Yr													
1	22.00	33.33	20.00	14.28	10.00	5.00	3.75	33.00	20.00				
2	22.00	44.44	32.00	24.49	18.00	9.50	7.22	45.00	32.00				
3	22.00	14.82	19.20	17.49	14.40	8.55	6.68	22.00	24.00				
4	22.00	7.41	11.52	12.49	11.52	7.69	6.18		16.00				
5	22.00		11.52	8.93	9.22	6.93	5.71		8.00				
6	22.00		5.76	8.93	7.37	6.23	5.28						
7	22.00			8.93	6.55	5.90	4.89						
8	22.00			4.46	6.55	5.90	4.52						
9	22.00				6.55	5.90	4.46						
10	22.00				6.55	5.90	4.46						
11	22.00				3.29	5.90	4.46						
12	22.00					5.90	4.46						
13	22.00					5.90	4.46						
14	22.00					5.90	4.46						
15	22.00					5.90	4.46						
16	22.00					3.00	4.46						
17	22.00						4.46						
18	22.00						4.46						
19	22.00						4.46						
20	22.00						4.46						
21	22.00						2.25						
22	22.00												
23	22.00												
24	22.00												
25	22.00												
26	22.00												
27	22.00												
28	22.00												
29	22.00												
30	22.00												
31	22.00												
32	22.00												
33	22.00												
34	22.00												
35	22.00												
36	22.00												
37	22.00												
38	22.00												
39	22.00												
40	22.00												
41													

Table 17.18

LIFE CYCLE COST ANALYSIS
for
The Horsefly Heating District
Klamath Falls, Oregon
April 28, 1987

Project Cost (Pv)	1,759.954	Benefit Cost Ratio		0.39 to 1
Equity Financing (Pv)	295,078	Return on Equity		14.825%
Debt Financing (Pv)	2,100,526	Simple Payback	8 years	5 months
Net Present Value	-1,068,686	Discounted Payback	12 years	2 months

	1 Natural Gas Present System	2 Electricity Present System	3 Property Tax & Insurance Present System	4 O & M Present System	5 Hot Water Revenues Proposed System
YEAR ZERO AMOUNT	50,500	150,000	2,500	15,000	220,000
1988	0	0	0	0	0
1989	0	0	0	0	0
1990	0	0	0	0	0
1991	0	0	0	0	0
1992	67,964	190,532	2,693	18,250	298,599
1993	72,177	202,155	2,734	18,980	317,411
1994	76,652	214,486	2,775	19,739	337,408
1995	82,018	227,570	2,816	20,529	358,665
1996	87,759	241,452	2,858	21,350	381,261
1997	93,902	256,180	2,901	22,204	405,280
1998	100,476	271,807	2,945	23,092	430,813
1999	107,509	288,387	2,989	24,015	457,954
2000	115,250	305,979	3,034	24,976	486,805
2001	123,548	324,644	3,079	25,975	517,474
2002	132,443	344,447	3,126	27,014	550,075
2003	141,979	365,458	3,172	28,095	584,729
2004	152,201	387,751	3,220	29,219	621,567
2005	163,160	411,404	3,268	30,387	660,726
2006	174,907	436,500	3,317	31,603	702,352
2007	187,501	463,126	3,367	32,867	746,600
TOTALS:	1,879,446	4,931,880	48,296	398,293	7,857,719

Table 17.19

**LIFE CYCLE COST ANALYSIS
for
The Horsefly Heating District
Klamath Falls, Oregon
April 28, 1987**

	6 Natural Gas Present System	7 Electricity Present System	8 Geothermal Proposed System	9 Property Tax & Insurance Proposed System	10 O & M Present System
YEAR ZERO AMOUNT	-400	-100,000	-10,000	-3,000	-25,000
1988	0	0	0	0	0
1989	0	0	0	0	0
1990	0	0	0	0	0
1991	0	0	0	0	0
1992	538	127,022	13,070	3,232	30,416
1993	572	134,770	13,788	3,280	31,633
1994	607	142,991	14,547	3,330	32,898
1995	650	151,713	15,347	3,379	34,214
1996	695	160,968	16,191	3,430	35,583
1997	744	170,787	17,081	3,482	37,006
1998	796	181,205	18,021	3,534	38,486
1999	852	192,258	19,012	3,587	40,026
2000	913	203,986	20,058	3,641	41,627
2001	979	216,429	21,161	3,695	43,292
2002	1,049	229,631	22,325	3,751	45,024
2003	1,125	243,639	23,553	3,807	46,825
2004	1,206	258,501	24,848	3,864	48,698
2005	1,292	274,269	26,215	3,922	50,645
2006	1,385	291,000	27,656	3,981	52,671
2007	1,485	308,751	29,178	4,041	54,778
TOTALS:	14,887	3,287,920	322,050	57,955	663,822

Table 17.20

**LIFE CYCLE COST ANALYSIS
for
The Horsefly Heating District
Klamath Falls, Oregon
April 28, 1987**

	11 Equipment Replacement Proposed System	12 Equipment Depreciation Proposed System	13 Debt Service Bank Loan and/or Bond Issue	14 Interest Charges on Financing	15 Net Operating Income
YEAR ZERO AMOUNT					
1988	0	0	159,247	132,000	-132,000
1989	0	0	159,247	129,548	-129,548
1990	0	0	159,247	126,875	-126,875
1991	0	0	224,345	163,961	-163,961
1992	0	138,544	224,345	158,276	106,346
1993	0	173,015	224,345	152,276	104,346
1994	0	156,664	224,345	145,243	154,780
1995	0	144,215	224,345	137,790	204,289
1996	0	134,727	224,345	129,633	253,454
1997	0	35,301	224,345	120,705	395,362
1998	0	33,252	224,345	110,932	442,907
1999	0	28,558	224,345	100,236	496,326
2000	0	23,875	224,345	88,528	553,416
2001	0	23,875	159,247	75,713	609,576
2002	735,962	179,888	159,247	68,195	507,242
2003	0	171,663	60,000	60,000	572,823
2004	0	167,913	60,000	60,000	628,930
2005	0	164,163	1,060,000	60,000	688,439
2006	0	164,163	0	0	807,822
2007	0	0	0	0	1,035,229
TOTALS:	735,962	1,739,815	4,219,688	2,019,688	7,009,498

Table 17.21

LIFE CYCLE COST ANALYSIS
for
The Horsefly Heating District
Klamath Falls, Oregon
April 28, 1987

	16 Percentage Depletion Allowance	17 Net Income Before Taxes	18 Federal and State Income Tax	19 Net Income After Taxes	20 Add Depreciation & Depletion
YEAR ZERO AMOUNT					
1988	0	-132,000	0	-132,000	0
1989	0	-129,548	0	-129,548	0
1990	0	-126,875	0	-126,875	0
1991	0	-163,961	0	-163,961	0
1992	53,471	53,470	0	53,470	192,015
1993	52,173	52,173	0	52,173	225,187
1994	74,230	80,550	0	80,550	230,894
1995	78,906	125,383	0	125,383	223,121
1996	83,877	169,576	0	169,576	218,604
1997	89,162	306,200	37,460	268,741	124,463
1998	94,779	348,128	174,064	174,064	128,030
1999	100,750	395,576	197,788	197,788	129,308
2000	107,097	446,319	223,160	223,160	130,972
2001	113,844	495,732	247,866	247,866	137,719
2002	121,016	386,226	193,113	193,113	300,904
2003	128,640	444,183	222,091	222,091	300,303
2004	136,745	492,185	246,092	246,092	304,658
2005	145,360	543,079	271,540	271,540	309,523
2006	154,517	653,305	326,652	326,652	318,680
2007	164,252	870,977	435,488	435,488	164,252
TOTALS:	1,698,819	5,310,679	2,575,315	2,735,365	3,438,634

Table 17.22

LIFE CYCLE COST ANALYSIS
for
The Horsefly Heating District
Klamath Falls, Oregon
April 28, 1987

	21 Net Operating Cash Flow After Taxes	22 Cumulative Cash Flow	23 Discounted Cash Flow	24 Cumulative Discounted Cash Flow	25 Discounted Cash Flow at ROR
YEAR ZERO AMOUNT					
1988	-159,247	-159,247	-144,770	-144,770	-138,687
1989	-159,247	-318,494	-131,609	-276,379	-120,782
1990	-159,247	-477,741	-119,645	-396,024	-105,188
1991	-224,345	-702,087	-153,231	-549,255	-129,056
1992	179,416	-522,671	111,403	-437,852	89,885
1993	205,068	-371,603	115,756	-322,096	89,473
1994	232,342	-85,260	119,228	-202,868	88,285
1995	261,949	175,688	122,201	-80,667	86,684
1996	293,468	470,156	124,459	43,792	84,576
1997	289,563	759,720	111,639	155,431	72,677
1998	188,681	948,401	66,132	221,563	41,243
1999	202,987	1,151,388	64,678	286,241	38,641
2000	218,315	1,369,702	63,238	349,479	36,194
2001	302,051	1,671,753	79,539	429,018	43,611
2002	-332,998	1,338,756	-79,717	349,301	-41,872
2003	522,395	1,861,150	113,688	462,990	57,206
2004	550,750	2,411,900	108,963	571,953	52,525
2005	-418,938	1,992,963	-75,350	496,603	-34,796
2006	645,333	2,638,296	105,517	602,120	46,679
2007	599,740	3,238,036	89,148	691,268	37,781
TOTALS:	3,238,036		691,268		295,078

Table 17.23

LIFE CYCLE COST ANALYSIS
for
The Horsefly Heating District
Klamath Falls, Oregon
April 28, 1987

Energy forecast used for this report: 1986 Sample Energy Forcast.* This report is for a taxable entity. Dollar values rounded to the nearest: 1

Capital investment data:

Year Of Initial Capital Investment	(example: 1988)	1988
Year Project Will Be In Production	(example: 1990)	1992
Project Life In Years	(enter a whole number from 1 to 40)	20
Economic Inflation Rate	(enter as a whole number e.g. 7)	4%
Discount Rate	(enter as a whole number 3.g. 12)	10%

Financing:

Beginning of Year	1992	1991	1990	1989	1988
Pr Jt Cost	....	500,000	300,000	700,000	500,000
Equity	....	60,000	....	....	250,000
Bank Loan	....	400,000	....	....	800,000
Life	....	10	....	....	15
(APR)	...........%	10%	...........%	...........%	9%
Bond Issue	....	....	....	....	1,000,000
Life	....	....	....	....	18
(APR)	...........%	...........%	...........%	...........%	6%
Invested	....	1,157	241,366	642,765	1,390,752
(APR)	...........%	8.5%	10%	9%	8%

Investment EOY 1992: 1,255 Total Project Cost (NPV): 1,759,954

Year Zero Annual Costs:	Present System	Proposed System
Conventional Energy:		
Natural Gas	50,500	400
Fuel Oil	....	....
Propane	....	....
Coal	....	....
Electricity	150,000	100,000
Other Energy:		
Geothermal	....	10,000
Solar	....	....
Nuclear	....	....
Biomass	....	....
Waste Heat	....	....
Insurance	2,500	3,000
Inflation Rate		1.5%
Operation and Maintenance	15,000	25,000
Inflation Rate		4%

Table 17.24

Year Zero Annual Cost:	Present System	Proposed System
Sales of Energy:		
Steam.......:	....	....
Hot Water.......:	....	200,000
Tipping Fees.......:	....	....

Equipment Purchases:	Class 1	Class 2	Class 3	Class 4	Class 5
Initial Cost:	500,000	250,000	120,000	90,000	....
Yr. Purchased:	1988	1989	1990	1990	....
Inflation/Yr:	4%	.........%	5%	.........%	6%
Actual Salvage.:	50,000	....	20,000	....	....
Economic Life:	10	25	18	20	....

Table 17.25

Sample Energy Forecast*
Forecast Energy Inflation Rates

1986

Yr	1 Natu	2 Fuel	3 Prop	4 Coal	5 Elec	6 Geot (1)	7 Sola (2)	8 Nucl (3)	9 Biom (4)	10 Wast (5)	11 Stea (1)	12 Hot (2)	13 Tipp (3)
86	0.020	0.019	0.019	0.020	0.009	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
87	0.020	0.019	0.019	0.020	0.009	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
88	0.020	0.019	0.019	0.020	0.009	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
89	0.020	0.019	0.019	0.020	0.009	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
90	0.022	0.019	0.019	0.020	0.009	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
91	0.022	0.019	0.019	0.020	0.009	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
92	0.022	0.019	0.019	0.020	0.009	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
93	0.022	0.019	0.019	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
94	0.022	0.019	0.019	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
95	0.030	0.019	0.019	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
96	0.030	0.019	0.019	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
97	0.030	0.019	0.019	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
98	0.030	0.019	0.019	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
99	0.030	0.019	0.019	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
00	0.032	0.018	0.018	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
01	0.032	0.018	0.018	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
02	0.032	0.018	0.018	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
03	0.032	0.018	0.018	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
04	0.032	0.018	0.018	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023
05	0.032	0.018	0.018	0.020	0.021	0.015	0.004	0.001	0.005	0.010	0.002	0.023	0.023

Table 17.26

LIFE CYCLE COST ANALYSIS
for
The Horsefly Heating District
Klamath Falls, Oregon
April 28, 1987

Project Cost (Pv)	1,759.954	Benefit Cost Ratio			0.72 to 1
Equity Financing (Pv)	295,078	Return on Equity			18.737%
Debt Financing (Pv)	2,100,526	Simple Payback		8 years	5 months
Net Present Value	-500,433	Discounted Payback		11 years	0 months
Discount Rate 10.00%					

	1 Natural Gas Present System	2 Electricity Present System	3 Insurance Present System	4 O & M Present System	5 Hot Water Revenues Proposed System
YEAR ZERO AMOUNT	50,500	150,000	2,500	15,000	220,000
1988	0	0	0	0	0
1989	0	0	0	0	0
1990	0	0	0	0	0
1991	0	0	0	0	0
1992	67,964	190,532	2,693	18,250	298,599
1993	72,177	202,155	2,734	18,980	317,411
1994	76,652	214,486	2,775	19,739	337,408
1995	82,018	227,570	2,816	20,529	358,665
1996	87,759	241,452	2,858	21,350	381,261
1997	93,902	256,180	2,901	22,204	405,280
1998	100,476	271,807	2,945	23,092	430,813
1999	107,509	288,387	2,989	24,015	457,954
2000	115,250	305,979	3,034	24,976	486,805
2001	123,548	324,644	3,079	25,975	517,474
2002	132,443	344,447	3,126	27,014	550,075
2003	141,979	365,458	3,172	28,095	584,729
2004	152,201	387,751	3,220	29,219	621,567
2005	163,160	411,404	3,268	30,387	660,726
2006	174,907	436,500	3,317	31,603	702,352
2007	187,501	463,126	3,367	32,867	746,600
TOTALS:	1,879,446	4,931,880	48,296	398,293	7,857,719

Table 17.27

LIFE CYCLE COST ANALYSIS
for
The Horsefly Heating District
Klamath Falls, Oregon
April 28, 1987

	6 Natural Gas Present System	7 Electricity Present System	8 Geothermal Proposed System	9 Insurance Proposed System	10 O & M Operating Cash Flow
YEAR ZERO AMOUNT	-400	-100,000	-10,000	-3,000	-25,000
1988	0	0	0	0	0
1989	0	0	0	0	0
1990	0	0	0	0	0
1991	0	0	0	0	0
1992	538	127,022	13,070	3,232	30,416
1993	572	134,770	13,788	3,280	31,633
1994	607	142,991	14,547	3,330	32,898
1995	650	151,713	15,347	3,379	34,214
1996	695	160,968	16,191	3,430	35,583
1997	744	170,787	17,081	3,482	37,006
1998	796	181,205	18,021	3,534	38,486
1999	852	192,258	19,012	3,587	40,026
2000	913	203,986	20,058	3,641	41,627
2001	979	216,429	21,161	3,695	43,292
2002	1,049	229,631	22,325	3,751	45,024
2003	1,125	243,639	23,553	3,807	46,825
2004	1,206	258,501	24,848	3,864	48,698
2005	1,292	274,269	26,215	3,922	50,645
2006	1,385	291,000	27,656	3,981	52,671
2007	1,485	308,751	29,178	4,041	54,778
TOTALS:	14,887	3,287,920	322,050	57,955	663,822

Table 17.28

**LIFE CYCLE COST ANALYSIS
for
The Horsefly Heating District
Klamath Falls, Oregon
April 28, 1987**

	11 Equipment Replacement Proposed System	12 Debt Service Bank Loan and/or Bond Issue	13 Net Operating Cash Flow	14 Cumulative Cash Flow
YEAR ZERO AMOUNT				
1988	0	159,247	-159,247	-159,247
1989	0	159,247	-159,247	-318,494
1990	0	159,247	-159,247	-477,741
1991	0	224,345	-224,345	-702,087
1992	0	224,345	179,416	-522,671
1993	0	224,345	205,068	-317,603
1994	0	224,345	232,342	-85,260
1995	0	224,345	261,949	176,688
1996	0	224,345	293,468	470,156
1997	0	224,345	327,023	797,179
1998	0	224,345	362,745	1,159,924
1999	0	224,345	400,775	1,560,699
2000	0	224,345	441,474	2,002,174
2001	0	159,247	549,917	2,552,091
2002	735,962	159,247	-183,177	2,368,914
2003	0	60,000	744,486	3,113,400
2004	0	60,000	796,843	3,910,243
2005	0	1,060,000	-147,398	3,762,844
2006	0	0	971,985	4,734,830
2007	0	0	1,035,229	5,770,058
TOTALS:	735,962	4,219,688	5,770,058	

Table 17.29

**LIFE CYCLE COST ANALYSIS
for
The Horsefly Heating District
Klamath Falls, Oregon
April 28, 1987**

	15 Discounted Cash Flow	16 Cumulative Discounted Cash Flow	17 Discounted Cash Flow at ROR
YEAR ZERO AMOUNT			
1988	-144,770	-144,770	-134,118
1989	-131,609	-276,379	-112,954
1990	-119,645	-396,024	-95,130
1991	-153,231	-549,255	-112,870
1992	111,403	-437,852	76,022
1993	115,756	-322,096	73,180
1994	119,228	-202,868	69,829
1995	122,201	-80,667	66,304
1996	124,459	43,792	62,561
1997	126,081	169,874	58,713
1998	127,140	297,014	54,850
1999	127,699	424,713	51,037
2000	127,879	552,592	47,349
2001	144,810	697,403	49,673
2002	-43,851	653,552	-13,935
2003	162,022	815,574	47,699
2004	157,651	973,225	42,997
2005	-26,511	946,714	-6,698
2006	158,927	1,105,641	37,201
2007	153,880	1,259,521	33,369
TOTALS:	1,259,521		295,078

GLOSSARY OF ENGINEERING COST ANALYSIS TERMS

Accelerated cost recovery system (ACRS) - A depreciation system developed in 1987 which provides more rapid cost recovery than straight line depreciation.

Annual effective interest rate - The actual or true annual interest rate that indicates the exact amount of interest paid per year by converting nominal or annual percentage rates to the actual dollar amount of interest annually.

Annual equivalent cost - The amount of an equal end-of-year payment series that is equivalent to present or future value when the interest rate is considered.

Annual percentage rate (APR) - The interest rate per year without considering the effect of compounding followed by the number of compounding periods per year.

Annuity due - An equal series of beginning-of-the-period payments.

Arithmetic gradient - An end-of-period payment series that increases each period by a constant amount.

Asset Depreciation Range (ADR) - The life of an asset as fixed by the Internal Revenue Code.

Benefit-cost ratio - The ratio that results in dividing the present value of all benefits of a project by the present value of all the costs of that same project.

Book value - The cost basis of an asset less the accumulated depreciation.

Business tax credit - A direct credit to the tax liability of a firm because of the purchase or operation of assets that are eligible for business tax credits under the current tax law.

Capital recovery - The process of charging periodic end-of-period payment series that is equivalent to the initial asset cost less the expected salvage value at the end of its economic life.

Capital recovery factor - The factor used to calculate the end-of-period payment series that is equivalent to the cost of an asset less its future salvage value based on the compounded interest rate.

Cash flow - 1. The actual flow of dollars into and out of the operation of a capital venture. 2. The flow of revenues less operating costs plus depreciation from a given capital venture.

Compound amount factor - 1. The future value of a lump sum considering time and the compound interest rate. 2. The future value of a payment series considering time and the compound interest rate.

Compound interest - Interest that is calculated and added to the initial amount such that future interest earnings will be accumulated based on the total amount including interest.

Cost of capital - The costs incurred because of borrowing money. Normally expressed as an interest rate.

Depletion - A method of depreciation applying to depletable resources such as geothermal, coal, timber, oil, and natural gas.

Depreciation - A method of expensing the decrease in value of an asset over its life span and charging these costs to operations.

Discounted cash flow - Evaluating the annual costs and revenues associated with a venture over the years of its life at some interest rate to arrive at a present value of these cash flows.

Discounted payback period - The time period required (considering the time value of money) for the revenues or benefits of a project equal the capital investment and operating costs of that project.

Discount rate - The interest rate used to evaluate the cash flows resulting from the operation of a project.

Double declining balance depreciation - A method of accelerated depreciation that applies twice the straight line rate multiplied by the book value of the asset.

Economic life - The period of time from installation to retirement of an asset that minimizes the cost of buying, installing, operating and salvaging that asset.

Energy tax credit - A direct credit to the tax liability of a firm because of the purchase or operation of assets that are eligible for energy tax credits under the current tax law.

Equipment cost basis - The adjusted cost of an asset due to additional installation costs or subtracted tax credits claimed which alter the amount of depreciation which may be claimed.

Equipment replacement life - The period of time that will elapse from initial installation until a piece of equipment is either obsolete or worn out and requires replacement.

Future value/future work - The equivalent value at a designated future date of previous cash flows evaluated over time at a compound interest rate.

Geometric gradient - An end-of-period payment series that increases each period by the exact same percentage.

Inflation - An increase in prices in general usually caused by a decline in the purchasing power of a monetary system.

Interest - The periodic cost charged for borrowing money.

Internal rate of return - The interest rate earned by investing in a particular venture and receiving cash flows as a result of that investment without regard to any other investments.

Investment tax credit - A direct credit to the tax liability of a firm because of the purchase of assets that are eligible for investment tax credits under the current tax law.

Modified Accelerated Cost Recovery System (MACRS) - The current method of depreciation in which property class life and decpreciation methods are set by law, and salvage value is treated as zero.

Marginal cost - The amount of cost associated with an increase in output or with a particular project.

Marginal tax rate - The amount of taxes (expressed as a %) because of an increase in output or the income of a particular project.

Net present value - The equivalent value at time zero (today) of the net difference between future cost and revenue cash flows evaluated (discounted) over time at a compound interest rate.

Nominal interest rate - The interest rate per year without considering the effect of compounding followed by the number of compounding periods per year.

Ordinary annuity - A stream of equal end-of-period payments.

Opportunity cost - The cost of not taking advantage of an investment opportunity because of limited capital, resources, or other conflicting circumstances and thereby losing the earnings or benefits associated with that investment.

Present value/present worth - The equivalent value at time zero (today) of future cash flows evaluated (discounted) over time at a compound interest rate.

Rate of return (ROR) - The compound interest rate that will cause the present value of all the costs and benefits associated with a project to equal exactly zero.

Salvage value - 1. The value obtained from an asset when sold, scrapped or retired from service. 2. The current fair market value of a piece of equipment.

Sensitivity - The amount of effect or impact experienced by changing one of the input variables of a project undergoing an engineering economic study in order to alter the acceptability or rejectability of that project.

Simple interest - Interest that is accrued periodically on only the original amount of a loan or investment, and that is not compounded.

Simple payback period - The time period required (without considering the time value of money) for the revenues or benefits of a project to equal the capital investment and operating costs of that project.

Sinking fund - A stream of equal end-of-period payments set aside in an investment fund for the purpose of replacing an asset or making a balloon payment on a debt.

Straight line depreciation - A method of depreciation in which the equipment cost basis of an asset is recovered in equal amounts over its taxable life based on time, units of output, miles driven, or hours of operation.

Sum-of-the-years-digits depreciation (SYD) - A method of accelerated depreciation in which the equipment cost basis of an asset is multiplied each year by the remaining years of life divided by [(n times n+1) divided by 2] to arrive at a depreciation charge for that year (where n equals the taxable life of the asset).

Sunk cost - A capital cost already incurred that cannot be recovered and is not to be considered or evaluated in making current decisions.

Time value of money - The future, present or annual equivalent value of cash flows considering the compound interest rate of borrowed money, the expected rate of earnings on invested capital, or the required rate of return on the capital investment required for a project.

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