MATH 341 : Linear Algebra I **(4-0-4)** 04/27/15

Catalog Description: The study of vectors and matrices in Euclidean space, their geometric interpretations and application to systems of equations. Includes linear independence of vectors, basis and dimension, introduction to linear transformations, eigenvalues and eigenvectors, diagonalization, determinants.

Course Objectives:

1. Solve systems of linear equations.
2. Understand vectors and their algebra in $\mathbb{R}^n$.
3. Understand the relationship of vectors to systems of equations.
4. Understand matrices and perform matrix algebra.
5. Use matrices to solve systems of equations.
6. Understand subspaces and their bases.
7. Understand and use linear transformations.
8. Understand eigenvalues and eigenspaces, diagonalization.

Learning Outcomes and Performance Criteria

1. Solve systems of linear equations.

   Core Criteria:

   (a) Solve systems of linear equations by all of the following methods: Gaussian elimination and Gauss-Jordan method.
   (b) Perform, by hand, elementary row operations to reduce a matrix to row echelon form.
   (c) Use a calculator or software to reduce a matrix to row echelon form or reduced row echelon form.
   (d) Use a calculator or software to solve a system of linear equations.
   (e) Solve applied problems using systems of equations.
   (f) Given the row echelon form of an augmented matrix for a system of equations, determine the rank of the coefficient matrix, and the leading variables and free variables of the system.
   (g) Given the row echelon form for a system of equations:
      - Determine whether the system has a unique solution, and give the solution if it does.
      - If the system does not have a unique solution, determine whether it is inconsistent (no solution) or dependent (infinitely many solutions).
      - If the system is dependent, give the general form of a solution and give some particular solutions.

   Additional Criteria:

   (h) Approximate a solution to a system of equations using Jacobi's method or the Gauss-Seidel method.

2. Understand vectors and their algebra in $\mathbb{R}^n$.

   Core Criteria:

(a) Find the distance between two points in $\mathbb{R}^n$. Find the vector from one point to another in $\mathbb{R}^n$; find the norm of a vector.
(b) Multiply vectors by scalars and add vectors, algebraically and geometrically.
(c) Find the dot product of two vectors, and the angle between two vectors. Find the projection of one vector onto another vector.

Additional Criteria:

(d) Recognize the rectangular equation of a plane in $\mathbb{R}^3$ and determine where the plane intersects each of the three axes. Give the rectangular equation of any one of the three coordinate planes.

3. Understand the relationship of vectors to systems of equations.

   Core Criteria:

   (a) Give the vector form (outer product form) of a system of equations.
   (b) Give the vector equation of a line through two points or the vector equation of a plane through three points.
   (c) Write the solution to a system of equations in vector form and determine the geometric nature of the solution.

4. Understand matrices and perform matrix algebra.

   Core Criteria:

   (a) Give the dimensions of a matrix. Identify a given entry, row or column.
   (b) Identify matrices as square, upper triangular, lower triangular, symmetric, diagonal. Give the transpose of a given matrix.
   (c) Multiply a vector by a matrix.
   (d) Give the identity matrix (for a given dimensional space) and describe its effect on a vector or another matrix.
   (e) Multiply two or more matrices "by hand" and identify when two matrices can not be multiplied together.
   (f) Model an applied situation with a matrix.

   Additional Criteria:

   (g) Determine whether a matrix is a projection matrix, rotation matrix, or neither, by its action on a few vectors.
   (h) Find a projection or rotation matrix.
   (i) Give the geometric or algebraic representations of the inverse or square of a rotation.

5. Use matrices to solve systems of equations.

   Core Criteria:

   (a) Express a system of equations as a coefficient matrix times a vector, equaling another vector.
   (b) Use the inverse matrix to solve a system of equations.
   (c) Find the determinant of a matrix by hand. Use a calculator or software to find the determinant of a matrix.
   (d) Use the determinant to decide whether a system of equations has a unique solution.

(e) Use LU-factorization to solve a system of equations, given the LU-factorization of its coefficient matrix.

(f) Obtain the LU-factorization of a matrix.

6. Understand subspaces and their bases.

Core Criteria:

(a) Describe the span of a set of vectors in $\mathbb{R}^2$ or $\mathbb{R}^3$ geometrically.

(b) Determine whether a vector $\mathbf{w}$ is in the span of a set $\{\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_k\}$ of vectors. If it is, write $\mathbf{w}$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_k$.

(c) Determine whether a set $\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_k$ of vectors is linearly independent or linearly dependent.

(d) Determine whether a subset of $\mathbb{R}^n$ is a subspace. If so, prove it. If not, give an appropriate counterexample.

(e) Determine whether a given set of vectors is a basis for a given subspace. Give a basis for, and the dimension of, a subspace.

(f) Determine whether a vector is in the column space or null space of a matrix.

(g) Find the dimension of, and basis for, the column space and null space of a given matrix.

Additional Criteria:

(h) Find the dimension of, and basis for, the row space of a given matrix.

7. Understand and use linear transformations.

Core Criteria:

(a) Determine whether a given transformation from $\mathbb{R}^m$ to $\mathbb{R}^n$ is linear. If it isn't, give a counterexample. If it is, demonstrate this algebraically.

(b) Give the standard matrix representation of a linear transformation.

(c) Find the composition of two linear transformations.

(d) For a transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, show the graphical differences between (a) $T(c\mathbf{u})$ and $cT\mathbf{u}$, and (b) $T(\mathbf{u} + \mathbf{v})$ and $T\mathbf{u} + T\mathbf{v}$.

(e) Find matrices that perform combinations of dilations, reflections, rotations and projections.

Additional Criteria:

(f) Give the coordinates of a vector $\mathbf{v}$ with respect to a basis $\mathcal{B}$. Find them algebraically or geometrically.

(g) Give the matrix that changes a vector in $\mathbb{R}^n$ to its representation with respect to another basis $\mathcal{B}$.

8. Understand eigenvalues and eigenspaces, diagonalization.

Core Criteria:

(a) Determine whether a given vector is an eigenvector for a matrix. If it is, find the corresponding eigenvalue.

(b) Find the characteristic polynomial and use it to find the eigenvalues of the matrix.

(c) Find complex eigenvalues for a 2×2 matrix.

(d) Give the eigenspace E_k corresponding to an eigenvalue λ_k of a matrix.

(e) Determine whether a matrix is diagonalizable.

(f) Diagonalize a matrix. Show the forms of the matrices P and D from $P^{-1}AP = D$.

(g) Present one application that requires the use of eigenvalues and eigenvectors (e.g. ranking, population models, Markov chains, adjancency matrices).

Additional Criteria:

(h) Determine the algebraic and geometric multiplicities of an eigenvalue.

(i) Write a system of linear differential equations in matrix-vector form. Write the initial conditions in vector form.

(j) Solve a system of two linear differential equations. Solve an initial value problem for a system of two linear differential equations.