A NEW MODEL FOR MATLAB INSTRUCTION AT OIT

Prepared by:
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A Human Brain in Matlab
Source: http://www.math.mcgill.ca/keith/surfstat/

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December 4, 2008

Ms. Jessamyn Schnackenberg
Oregon Institute of Technology
3201 Campus Drive
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Dear Ms. Schnackenberg:

Enclosed is my recommendations report on developing a new model for Matlab instruction at OIT. The purpose of the report is to identify problems with the current model of teaching Matlab software to OIT students, and to introduce a new teaching model to address these problems. This report makes three main recommendations for the development of a new teaching model: creating a website for Matlab instruction, offering a 1-credit Matlab course alongside the calculus sequence using the website, and using the website to increase the number of demonstrations and visualizations for students. The first section of the report discusses problems with the current teaching model and the second section discusses how the new model will solve these problems.

The current method of teaching Matlab to OIT students has several disadvantages for students and instructors. Among these are a lack of structure and consistency in the presentation and use of the software, a lack of continuity in the use of Matlab during student's time at OIT, and too little use of Matlab as a device for illustrating and reinforcing mathematical concepts. In order to create an effective teaching model, research was conducted to determine what successful strategies were employed by other institutions and a survey of over 300 OIT students was conducted to assure local student body's needs were addressed.

I would like to thank the following people for their generous support and guidance: Tiernan Fogarty who was my technical reader, Cristina Negoita who posted my survey on the OIT Mathematics Department website, Connie Dernbach and Erin Foley who helped distribute the survey through OIT email. I would also like to thank the OIT Mathematics Department faculty for providing helpful feedback about improving my survey and I am especially grateful to all the students who provided valuable feedback by participating in the survey.

Please present these recommendations to the OIT Mathematics Department, so that they can begin implementing them. Research and student feedback indicates that problems exist with the current model of teaching Matlab to OIT and the recommendations provided below present a way to address these problems.

Sincerely,

Brian McKenzie
Student in Applied Mathematics
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EXECUTIVE SUMMARY

There are several problems with the current method of teaching Matlab to students at OIT. A survey of OIT students conducted for this report found that the current model is deficient in: incorporating Matlab instruction into mathematics classes, providing clear and specific goals for learning Matlab, providing adequate help and assistance for learning Matlab, providing context for learning Matlab, and providing interactive demonstrations and visualizations using Matlab. As a result, many students learning Matlab at OIT have difficulties understanding why they are learning the software and how it is useful to their future. They are also placed at a disadvantage compared to students at other institutions where the software is used to effectively introduce and reinforce concepts through active visualizations and demonstrations.

This report recommends three main solutions to address the problems with the current teaching model. First, a website must be created to assist in teaching Matlab to OIT students. Second, the teaching of Matlab must be integrated into the core mathematics curriculum. This can be done by creating a 1-credit course that can be taken concurrently with the calculus sequence, and that will make use of the OIT Matlab website. Third, the website must be used to help relate mathematical material to the students' fields, and to demonstrate the usefulness of the material and Matlab to them.

The solutions recommended in this report attempt to address the problems of the current teaching model while disrupting the core curriculum as little as possible for those students not learning Matlab as part of their degree requirements. These recommendations make use of the findings published in academic journals by others who have attempted to solve similar problems at their learning institutions. The recommendations were also made on the basis of their ability to incorporate suggestions offered by the OIT students surveyed.

By following the recommendations above, the OIT Mathematics faculty can expect to see increased motivation and interest among students, increased appreciation for the software and concepts covered in lectures, and an increase in communications and critical thinking skills. It is recommended that the OIT Mathematics Department begin immediate work on creating the website, developing a 1-credit course to accompany the website, and developing strategies to increase the number of interactive visualizations and demonstrations using Matlab and the new website once it is created.

A NEW MODEL FOR MATLAB INSTRUCTION AT OIT

INTRODUCTION

This is a report on how Matlab is taught at Oregon Institute of Technology. This introduction discusses the purpose, scope, and format of the report.

PURPOSE OF THE REPORT

The purpose of the report is twofold. First, it seeks to identify problems with the current model for teaching Matlab at OIT in terms of the learning environment, course implementation, and pedagogy. Second, it explores ways to supplement or change the current teaching model using web-based technologies and alternative course structures.

SCOPE OF THE REPORT

The objectives of the report are to clarify what problems currently exist with Matlab instruction at OIT, and to show how a new model would address these problems. The cost of implementation of the new model and details about the time requirements remain to be explored later.

FORMAT OF THE REPORT

The report includes the following sections:
- Background on Matlab.
- Problems with the current teaching model.
- Solutions to address these problems.
- Conclusions.
- Recommendations.

MATLAB INSTRUCTION AT OIT:
PROBLEMS AND SOLUTIONS

This section first discusses, in detail, the problems associated with how Matlab is currently taught at OIT. Using research conducted by educators elsewhere and feedback from students here at OIT a new model for teaching the software is then introduced and discussed in detail. The statistics for the OIT student survey cited in this report can be found in the appendix.

BACKGROUND ON MATLAB

Matlab (short for "Matrix Laboratory") is a powerful software program that was first developed by engineers, mathematicians, and computer scientists in 1984 [1]. The software provides users with a productive computing environment, which includes comprehensive mathematics and graphics functions, and a powerful, high-level programming language [1].

According to [1], Matlab is used by over 1,000,000 professional engineers and scientists in areas such as technology, government research, and finance, and in more than 3,500 universities for research and instruction.

Matlab is similar to programs like Mathematica and Maple, but differs from them in significant ways. Mathematica and Maple have highly developed front ends (how users interact with the software) that allow the user to easily and intuitively perform symbolic manipulation. The downside of these two programs is that they can be computationally expensive and they have more complicated interfaces than the standard Matlab interface. Matlab, on the other hand, works almost exclusively with floating point arithmetic. This helps increase computational speed and, by conforming to IEEE standards, provides sufficient accuracy for most practical problems [2]. The Matlab interface is command-driven, which forces users to understand details about the program, and to master a good deal of syntax, but also allows the user to begin by using the basic command line interface, and then gradually adding more complex features as the need arises (Figure 1).
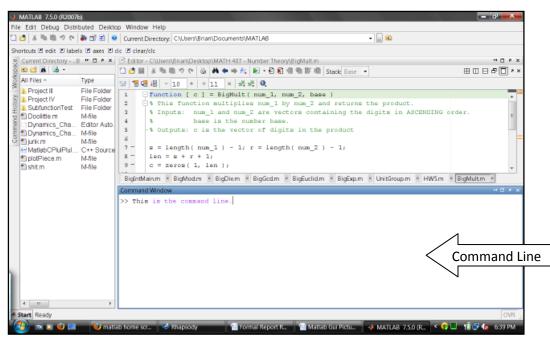
Figure 1
The Matlab Command Line Interface

Using the graphical user interface (GUI) capabilities of Matlab, one can write programs that give users more intuitive environments than the standard command-line interface, and obviate the need to learn syntax before using the software. Figure 2 shows a GUI that allows users to easily create graphs using the mouse and toolbar options. The syntax of the Matlab programming language is very similar to C/C++ and other procedural languages. Newer versions of Matlab also have object-oriented capabilities. Matlab can also be customized and expanded to meet the user's specific needs. For example, there is an add-on (referred to as a toolbox by Matlab users) for Matlab that allows for symbolic computing by using the Maple kernel.
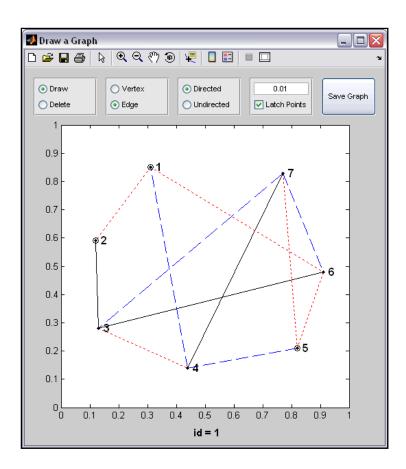
Figure 2
A Matlab GUI Illustrating Graph Theory
Source: http://webscripts.softpedia.com/script/Development-Scripts-js/Graph-GUI-37517.html

PROBLEMS WITH THE CURRENT TEACHING MODEL

The current method of teaching MATLAB to OIT students in MATH 221 and ad hoc in other classes has several disadvantages for students and teachers. This section discusses these problems in detail.

Lack of Centralization and Structure for Matlab Help and Instruction

As it stands, there is no central place where students learning Matlab at OIT can go, outside of the lectures in MATH 221, to get help when they have difficulties related to the software. They must try to find a time when their instructor is available or look for resources outside of OIT. There is also no comprehensive manual that students can access to supplement the class instruction. A survey was conducted of OIT students in which 90 percent of those surveyed indicated that having access to an online manual for Matlab would be helpful to students learning Matlab. In *Too much, too seldom,* Dr. Leone Burton discusses his study of a group first year undergraduate engineers at Chalmers University where Matlab had been recently introduced as part of the mathematics curriculum [3]. The 42 students interviewed by Dr. Burton

indicated that they felt it was necessary for students to have access to a comprehensive support manual for students learning Matlab for the first time. In *Matlab in first-year engineering mathematics,* Dr. Len Colgan of the University of South Australia discusses the how the production of such a manual was a critical part of his successful incorporation of Matlab into the mathematics program there [4].

The lack of a central environment for Matlab assistance and instruction also makes it difficult for students and faculty to share ideas and collaborate on projects related to the software. It also presents an obstacle to developing common goals and structure relating to how Matlab is introduced and used in mathematics classes at OIT. The resulting inconsistency can contribute to a general lack of clarity among students about why they are learning the software and when it is beneficial to do so. In the survey of OIT students 25 percent indicated that the goals for learning and using Matlab were not made clear to them when they were asked to do so. Another 40 percent said the goals and reasons were only somewhat clear. This lack of clarity can lead to decreased motivation and interest toward learning the software and the class in general. One OIT student provided the following feedback regarding their initial introduction to Matlab in MATH 254N. "I was sent to the computer lab with a set of instructions and told to 'play around with it' with no personal guidance whatsoever. By the time I was done 'playing around with it', I was so thoroughly frustrated and disgusted with it that I dropped the class."

Fragmented Learning

In MATH 221 students are given an overview of Matlab, but no topic is treated in detail. Students in the class are asked to complete a set of exercises that introduce the various aspects of Matlab. These include basic syntax, command-line interaction, graphing, and topics from programming. Although this teaching model is praiseworthy for its intent, it has some limitations and disadvantages that the new model seeks to address. First, the mathematical material up for discussion in this course is limited by the disparity of mathematical backgrounds among students in the class. Another problem is that many students will not see much of Matlab again until much later since it is not used a lot in the introductory calculus sequence. This lack of continuous exposure and reinforcement inhibits long-term retention, meaning that many students will have to relearn the material later.

Matlab is used by some OIT professors in classes outside MATH 221 such as linear algebra and differential equations, and in some of the later introductory calculus courses to solve specific problems, which gives certain students additional exposure to the software. Because of the large amount of material covered in OIT's math course, however, there is little time for instructors who choose this option to explain how the software works in any significant detail. This can result in students perceiving Matlab as a "black box" that provides solutions without contributing significantly to the students' understanding [3]. Several of the OIT students

surveyed commented on the dangers of using powerful software like Matlab without an understanding of why it works.

Underutilized Pedagogical Tool

Because of the breadth of material covered in the introductory calculus sequence, certain aspects of mathematics, especially developing skill at obtaining analytical solutions to typical problems are emphasized while other aspects are neglected. These neglected aspects often include the kind of mathematical experimentation and discovery that Matlab readily facilitates, and that can help students construct meaning from ideas and develop problem-solving skills [5].

Interactive visualizations and demonstrations, which can be produced using Matlab's powerful graphical and graphical user interface (GUI) capabilities, are not used as much as they could be to help introduce and reinforce traditional mathematical concepts. 70 percent of the OIT students surveyed said that they were somewhat interested or very interested in seeing more interactive visualizations and demonstrations of concepts covered in lectures using computers. In addition, Matlab is not used as effectively as it could be to increase students' communication skills, and to show how mathematics is applicable to their field. As noted in *An innovative learning model for computation in first year mathematics*, the latter can be especially helpful in keeping many non-math majors interested and motivated in the material covered in lecture [6]. 60 percent of the OIT students surveyed said demonstrations of how the lecture material was directly applicable to their field were not given often or were never given.

SOLUTIONS TO ADDRESS THE PROBLEMS

There are several options available for improving the way Matlab is taught at OIT that will address the problems mentioned above. The problems seen at OIT are not unique to the institution, and this report is not the first to look into these issues. Others have attempted to find ways to solve them at their learning institutions, and have published their results in academic journals such as the *International Journal of Mathematical Education in Science and Technology*. The following section makes use of this previous experience as well as OIT student feedback to develop a new model for teaching Matlab that will better serve OIT's students.

The solutions are organized into three main groups: providing a web site for Matlab instruction at OIT, creating a course that uses the web site to help integrate Matlab instruction into the core math curriculum, and increasing the use of Matlab as a tool for OIT instructors.

Provide a Website for Teaching Matlab

A major part of the solutions this report presents is the idea of creating a website devoted to Matlab instruction at OIT. Here I will discuss the envisaged components of the website, and the expected benefits from such a venture.

Teaching Modules In [6], the authors discusses how, by creating several teaching modules, they were able to successfully incorporate Matlab instruction into the core curriculum at the University of Southern Queensland. In order to provide a flexible, yet structured environment for students and instructors at OIT, a sequence of web-based modules would need to be developed for each class in which Matlab is to be used. These modules would be divided into two categories: core modules, and project modules. The core modules would be designed to provide guided, interactive learning of Matlab by using GUIs (Figure 3) and pre-written code (Figure 4) with accompanying explanations and directions.
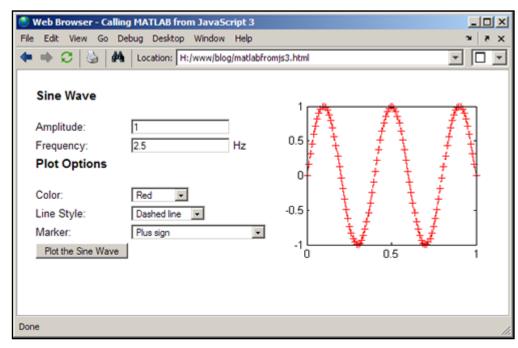

Figure 3
A GUI-Style Instruction Module
Source: http://blogs.mathworks.com/desktop/2008/02/25/

Figure 4
Prewritten Matlab Code with Explanations
Source: http://www.math.utah.edu/lab/ms/matlab/matlab.html

The goal of these core modules would be to gradually introduce new syntax and build enough knowledge of the software to complete the next project module. They would also serve as a place to give interactive visualizations and demonstrations of concepts from lectures. 40 percent of the OIT students surveyed said they found Matlab very useful in visualizing data and when asked how technology could be used to improve the learning experience at OIT several students indicated that visual presentation of concepts helped them better understand those concepts (reference to 3D graphics was especially common). Figure 5 shows an example of Matlab's ability to plot data for easy visualization.
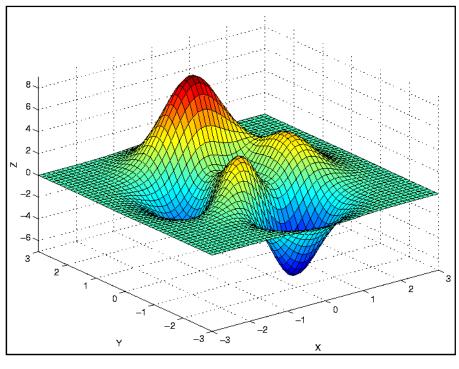
Figure 5
Surface Plot in Matlab

      The project modules would be more open-ended and designed with several objectives in mind. These would help show students how mathematics is applied to a particular field. This would be accomplished by creating a group of several project modules corresponding to the same topic. Each module in the group would deal with a specific area of application, and the student would then choose the one he or she found most appealing or applicable to his or her goals. Several of the OIT students responding to the survey indicated a desire to see more activities that would show how knowing the material they learn in the lectures is going to serve them in their future careers. The project modules would give instructors an opportunity to show them how and why mathematics is useful to their students.

      The project modules would also facilitate group activity among students if instructors chose to use the modules in this manner. Several of the sources consulted in researching this report indicated that students generally responded well to collaborative learning of Matlab, and showed how it can be beneficial to their overall education in areas such as communication, critical thinking, and general social training [3][6].

      Online Instruction Manual   Another necessary component of the website would be a comprehensive Matlab manual written by OIT professors for the specific needs of OIT students. This manual would include help on Matlab syntax, brief tutorials and examples to quickly refresh students on material when needed. The manual would also provide answers to

the most frequently asked questions (FAQ) and material relating to common error messages in Matlab. As Dr. Colgan discusses, such a manual not only presents factual information, but also "provides a bridge between the software and the core subject material covered in the textbooks and lectures" [4].

Discussion Forum The final component of the website would be a forum for users to communicate with one another about issues relating to Matlab. The forum would include a bulletin board for ongoing discussions where students could provide comments and helpful suggestions related to homework or projects. The responses to the OIT student survey indicate that many students are eager to explore new and innovative ways of learning and want to share their feelings about how OIT can best serve them and a discussion forum would provide them with an opportunity to do so.

Expected Benefits A website devoted to Matlab instruction at OIT would provide needed structure and consistency in how the software is introduced and used here. It would also provide a place where goals and reasons for learning Matlab could be made clear to the student. Once the modules are created they provide a convenient framework for teaching that any instructor can use without the need to "reinvent the wheel" each time. The modular design also lends itself well to expansion and revision when needed.

The site would serve as a central hub for students and professors to collaborate and share information with one another. This would be helpful in generating ideas for projects, efficiently distributing information, increasing teamwork and communication among students, and provide valuable feedback for instructors.

Having access to online assistance through the comprehensive help manual and FAQ would be a more efficient use of the students' time than having to search for someone at OIT to help them or to having to try to find assistance elsewhere. Having this information stored in a single repository where it could be easily accessed whenever needed would also reduce the need for instructors to cover the same material several times and would provide those who need to spend more time on the material to do so at their own pace.

Finally, the use of a website has the added benefit that all the material can be easily linked together so that the user can more easily see how the various aspects of Matlab and the various mathematical topics included are connected throughout their learning experience.

Offer Matlab Instruction Concurrently with the Calculus Sequence

In order to make use of the website and incorporate the instruction of Matlab into the core mathematics curriculum, an additional one-credit course would be developed for each of the courses in the introductory calculus sequence. These courses would be designed to be taken

concurrently with the calculus classes using modules tailored to the specific material covered in lecture. In this section I will outline the structure and expected benefits of these classes.

Course Structure   Each one-credit course would be based on completing eight modules by the students: six core modules, and two project modules per term. During each of the first three weeks of the term students would complete one core module. Each of these would introduce syntax and various aspects of Matlab through interactive tutorials. The mathematical subject matter covered in these modules would be chosen to reinforce the concepts covered in lecture. In the fourth week, the students would choose from a group of project modules, which they would then have two weeks to complete. During weeks six, seven, and eight, students would complete three more core modules. The final two weeks would be spent completing another project module.
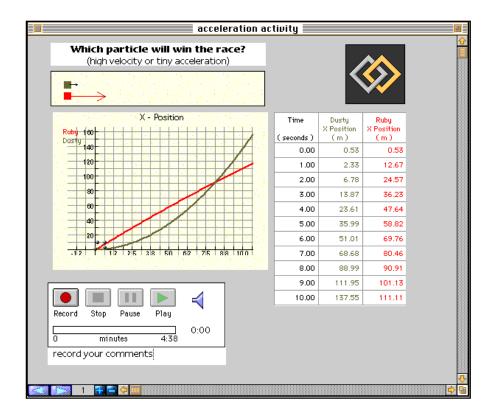
The grading of student work would be most easily accomplished by publishing a workbook similar to those currently used in MATH 221, which the students would purchase from the Paper Owl. The core modules would include intermittent prompts for the student to complete a portion of the workbook, and hints could be provided to some problems. For the project modules, the students would need to write a report following guidelines that would be included on the OIT Matlab website.

Expected Benefits Students who chose to learn Matlab in this more integrated fashion would benefit in several ways. A more gradual and continuous exposure to Matlab would provide more reinforcement of the material learned and give them more time to practice. This method of accumulating knowledge of the Matlab syntax and commands would also help reduce the learning curve that prevents some instructors from using the software extensively or at all. There would also be a likely increase in long-term retention among these students.

Instructors would benefit from knowing the students have a common mathematical background, and which topic they are currently working on. This would allow for a more focused and detailed presentation of mathematical concepts. It would also allow more mathematically sophisticated and accurate language to be used in the modules than is appropriate in a setting like MATH 221.

The integrated learning model would also be useful in helping students see connections within Matlab and in mathematics in general. As mentioned above, the website allows material to be easily linked, so that users can click on key words and be taken to other places in the modules and the manual where these terms are used. The modules can also be useful in presenting multiple representations of data and concepts, such as graphical, tabular, and schematic together. These different representations can be dynamically linked using Matlab, so that patterns between and among them become clearer to students. As discussed by the authors

of *Designing online mathematical investigation*, this sort of presentation can lead students to a higher level of understanding of mathematical concepts and how they are related [7]. Figure 6 is an example of data being simultaneously displayed in graphical, tabular, and schematic representations.

Figure 6
Multiple Representations of Data
Source: http://jime.open.ac.uk/98/6/roschelle-98-6-paper.html

The new courses would have the added benefit of being more convenient for many students than a traditional classroom lecture. Students could access modules as many times as they needed and at a time that was best for their schedule. The additional support and assistance given by the materials on the website would allow professors to take on a role of providing supplemental instruction on Matlab, and give motivated students an opportunity to begin to learn more on their own. Teaching assistants and tutors for mathematics could also be trained on the software to help students in person when necessary.

Increase the Use of Matlab as a Teaching Device by Instructors

One final way to address the problems with the current model of teaching Matlab to OIT students is to find ways to harness the software's power as a pedagogical tool for instructors.

This section discusses how Matlab can be used as an aid to instructors in presenting content and achieving didactic goals.

Designing and Setting Goals for the Modules   To ensure that the teaching modules serve OIT students in an efficient and pedagogically sound manner, the entire mathematics faculty must be involved in setting goals and standards for their design and implementation. As discussed in [7], a careful consideration of these issues is critical because "poorly chosen representations can have a detrimental effect on learning, not just a neutral one."

In terms of mathematical content, the faculty would need to determine what topics to cover in the modules. As the authors of *Calculus demonstrations using Matlab* discuss, there are many topics in introductory calculus classes such as limits, differentiation, integration, and related rates that lend themselves well to presentations and activities using Matlab [8]. The faculty would need to decide which topic would be best to include in the modules and which to leave out. The modules would also have to be designed so that relationships among the various topics are revealed.

Faculty would also need to provide input regarding the processes and aspects of Matlab used in the modules. Creating the correct balance of GUI, command-line, and programming for each module to meet goals would be challenging and require a commitment to improving and refining the modules over time. The website could be designed to provide statistics and feedback that would aid in this process.
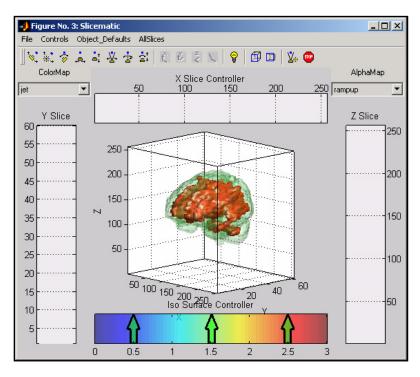
Increase Experimentation and Discovery in Curriculum   In addition to providing a means to reinforce and illustrate concepts covered in lecture, the modules provide an opportunity to show students the aspects of mathematics that are often neglected in introductory calculus courses. Certain modules could be designed to show how experimentation and discovery are important parts of mathematics. Students would be given an opportunity to apply the inductive and experimental methods used in other sciences to mathematics, and would be presented with a more complete picture of the subject. Jonathan Borwein notes, in *Mathematics by experiment: Plausible Reasoning in the 21ˢᵗ Century*, how this type of learning can be helpful in helping students in the following ways [5].

- Gaining insight and intuition.
- Discovering new patterns and relationships.
- Discovering underlying mathematical principles.
- Testing and especially falsifying conjectures.

Promote Teamwork and Increase Communication Skills The ability to work well in groups is cited regularly by employers as among the most important factors in deciding

whether to hire an applicant. The project modules provide means to increase the communication skills of OIT students. Allowing or requiring projects to be completed in teams would give students another opportunity to acquire experience and training in collaborative work.

Contextualize Mathematical Material   As mentioned above the project modules would be useful in placing the material covered in lecture into a context with which the student is familiar. The added motivation, interest, and appreciation that this type of project-based learning can produce are discussed in *Incorporating Matlab into university mathematics programs* [2]. Figure 7 shows Matlab being used to estimate white and gray matter in the human brain. An application of this type would help produce the desired effects mentioned above in students studying the life sciences.

Figure 7
Estimation White and Gray Matter with Matlab
Source: http://www.mathworks.com/company/pressroom/image_library/biotech.html

Several of the OIT students surveyed commented on this issue in the essay portion. When asked to provide ways technology could be used to improve the learning experience in mathematics classes, one student said "It would be great to get a hands on example of what we are or could be using the math for in our future careers" and another added "I think by showing how concepts apply to real life siituations [*sic*]. Most students think they will never use the math they learn."

CONCLUSIONS AND RECOMMENDATIONS

This section draws some conclusions and makes some recommendations based on the research conducted for this report.

CONCLUSIONS

As discussed in the section titled PROBLEMS WITH THE CURRENT TEACHING MODEL, there are several areas in which the current method of teaching Matlab to OIT students is lacking. These include not enough structure and consistency in the presentation and use of the software, lack of continuity in the use of Matlab during student's time at OIT, and too little use of Matlab as a device for illustrating and reinforcing concepts.

As demonstrated in the section titled SOLUTIONS TO ADDRESS THE PROBLEMS, there are several options available to solve these problems. The solutions offered in this report take advantage of web-based technology, and leave the option open for students not interested in learning Matlab or for those not required to learn Matlab to opt out of the new model and be unaffected by changes to the curriculum.

RECOMMENDATIONS

The following recommendations are based on the conclusions drawn above. It is recommended that the OIT Mathematics Department begin working on the following:

- Creating a website for Matlab instruction at OIT.
- Offer a 1-credit Matlab course concurrently with the calculus sequence using the website.
- Use the website to increase the number of demonstrations and visualizations for students.

These recommendations are intended to address the problems identified by OIT students, and are based on suggestions offered by these students, and by others who have tried to develop solutions to similar problems at other learning institutions.

REFERENCES

[1]   The MathWorks' Company Website, 1994-2008,
        http://www.mathworks.com/company/aboutus/.

[2]   Stanoyevitch, A., 2005, Incorporating Matlab into university mathematics programs,
        *Proceedings of KAIST International Symposium on Enhancing University Mathematics
        Teaching*, Daejeon, Korea, 12-16 May 2005. Available at
        http://www.csudh.edu/math/astanoyevitch/papers/no19_KAIST_05.pdf.

[3]   Burton, L., Falk, L., and Jarner, S., 2004, 'Too much, too seldom', *International Journal
        of Mathematical Education in Science and Technology*, **35**, 219-226.

[4]   Colgan, L., 2000, MATLAB in first-year engineering mathematics, *International Journal
        of Mathematical Education in Science and Technology*, **31**, 15-25.

[5]   Borwein, J., and Bailey, D., 2008, *MATHEMATICS BY EXPERIMENT: Plausible
        Reasoning in the 21st Century*, (AK Peters).

[6]   Tonkes, E. J., Loch, B. I., and Staces, A. W., 2005, An innovative learning model for
        computation in first year mathematics, *International Journal of Mathematical Education
        in Science and Technology*,**36**,751-759.

[7]   Gadanidis, G., Sedig, K., and Laing, H., 2004, Designing online mathematical
        investigation, *Journal of Computers in Mathematics and Science Teaching*, **23**, 275-298.

[8]   Dunn, P. and Harman, C., 2002, Calculus demonstrations using Matlab, *International
        Journal of Mathematical Education in Science and Technology*, **33**, 584-596.

APPENDIX