**MATH 342 :** Linear Algebra II **(3-0-3)** 03/07/11

**Catalog Description:** A continuation of the topics of MATH 341 to the setting of abstract vector spaces. Includes the study of orthogonality, inner product spaces, eigenvalues and eigenvectors, matrix decompositions and a more advanced study of linear transformations.

**Course Objectives:**

1. Use orthogonality in $\mathbb{R}^n$.
2. Understand vector spaces.
3. Use the inner product.

**Learning Outcomes and Performance Criteria**

1. Understand the significance of orthogonality in $\mathbb{R}^n$.

   Core Criteria:

   (a) Find the representation of a vector with respect to an orthogonal or orthonormal basis by projection.

   (b) Determine whether a set of vectors is orthogonal or orthonormal.

   (c) Given a basis for a subspace of $\mathbb{R}^n$, find a basis for the orthogonal complement of the space.

   (d) Given a basis for a subspace $W$ of $\mathbb{R}^n$ and a $\mathbf{v}$ in $\mathbb{R}^n$, find the unique orthogonal decomposition $\mathbf{v} = \mathbf{w} + \mathbf{w}^{\perp}$ with $\mathbf{w}$ in $W$ and $\mathbf{w}^{\perp}$ in $W^{\perp}$.

   (e) Apply the Gram-Schmidt process to a set of vectors; find the $QR$ factorization of a matrix.

   (f) Give the spectral decomposition of a symmetric matrix.

2. Understand and work with general vector spaces.

   Core Criteria:

   (a) Given a set of objects and definitions of addition and scalar multiplication,

      i. prove any of the properties of a vector space that hold,

      ii. give specific counterexamples to any properties that do not hold,

      iii. if there is a zero vector, determine what it is; if all vectors have inverses, give the general form of the inverse of a vector,

      iv. determine whether the set with those operations is a vector space.

   (b) Determine whether a subset of a vector space is a subspace.

   (c) Determine whether a set of vectors is linearly independent; if not, give one as a linear combination of the others.

   (d) Determine whether a set of vectors is a basis for a vector space; if not, tell why.

   (e) Give the representation of a vector with respect to a given basis.

   (f) Extend a set to a basis for a given space.

(g) Reduce a spanning set to a basis.

(h) Given the representation of a vector with respect to one basis, determine its representation with respect to another.

(i) Determine the change-of-basis matrix from one basis to another.

(j) Determine whether a given transformation is linear.

(k) Given the action of a linear transformation on basis vectors, find the linear transformation of any vector.

(l) Determine whether a given vector is in the kernel or range of a linear transformation. Describe the kernel and range of a linear transformation.

(m) Determine whether a given linear transformation is (a) one-to-one, (b) onto.

(n) Determine whether two vector spaces are isomorphic. If they are, give an isomorphism from one to the other.

(o) Determine the matrix of a linear transformation with respect to given bases.

3. Understand and work with the inner product.

   Core Criteria:

   (a) Determine whether a given operation on a vector space is an inner product.

   (b) Compute the inner product of two vectors, norm of a vector, distance between two vectors. Determine whether two vectors are orthogonal.

   (c) Apply the Gram-Schmidt process to a set of orthogonal vectors to obtain an orthogonal basis.

   (d) Compute the least squares line or parabola for a set of data points. Compute the least squares solution to a system of equations. Solve problems involving least squares approximation.

   (e) Find the standard matrix of the approximation onto a subspace; find the projection of a vector onto a subspace.

   (f) Find the singular values of a matrix; find the singular value decomposition of a matrix.