THERMAL RESPONSE TESTING OF GEOTHERMAL WELLS FOR DOWNHOLE HEAT EXCHANGER APPLICATIONS

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ABSTRACT

Accurate prediction of transient subsurface heat transfer is important in sizing downhole heat exchangers (DHEs) and making predictions of their thermal output, but quantification of these processes has been difficult and elusive in practice. As such, current DHE design methods rely on empirical data and rules of thumb. The work described in this paper makes use of so-called in-situ thermal response testing, in conjunction with a newly-adapted analytical solution that describes the coupled conductive and advective heat transport relevant to DHEs. The complex heat transfers within the well bore are described by a lumped thermal resistance parameter. A parameter estimation technique is applied to thermal response test data at a site in southern Oregon to quantify the average rock thermal conductivity, apparent average linear groundwater velocity, and wellbore thermal resistance. An example is given on use of the method to make DHE temperature output predictions over time of operation for an actual heating application.

INTRODUCTION

Accurate design tools for downhole heat exchangers (DHEs) in geothermal applications have remained elusive. This dilemma exists for essentially two main reasons: (1) lack of an easy-to-apply mathematical model that adequately describes heat transfer parameters relevant to DHEs, and (2) lack of a field test procedure to measure parameters for mathematical models. These reasons are intimately related, and detailed mathematical models are not applicable in practice if their solutions contain parameters that cannot be easily quantified in the field.

DHEs are unique in that they are characterized by numerous simultaneous heat transfer processes, namely: conduction through rock, advection due to regional groundwater flow, and natural convection of groundwater in the well bore. The design process is further complicated by the thermal resistance imposed by the DHE geometric configuration (i.e., pipe size, arrangement of pipes in the well bore, well completion and cased interval, presence of a convection promoter, and fluid flow within the DHE) and transient thermal loading applied to the DHE. Each of these processes is difficult to quantify in practice, and consequently, current DHE design methods rely on empirical data and rules of thumb.

The work described in this paper makes use of the so-called in-situ thermal response test, in conjunction with a newly-adapted analytical solution to describe the coupled conductive and advective heat transport relevant to DHEs to facilitate their design and predict their output. A key element of this approach is that it allows complex heat transfer processes within the well bore to be lumped into a single thermal resistance term. The thermal response test procedure is similar to that commonly conducted on closed-loop, grouted vertical borehole heat exchangers for use in geothermal heat pump applications, where a constant heat rate is applied to a circulating fluid stream in the DHE, and the inlet and outlet temperatures are recorded. The average rock thermal conductivity, apparent average linear groundwater velocity, and wellbore thermal resistance are estimated using a parameter estimation technique in conjunction with the analytical solution and thermal response test data.

BACKGROUND AND THEORETICAL CONSIDERATIONS

Culver and Reistad (1978) developed a design approach for DHEs that was centered around a so-called mixing ratio which was used to model convection cells in DHE well bores. This mixing ratio expressed the amount of groundwater leaving the well bore in proportion to new groundwater entering the well bore, and was used in conjunction with Darcy's Law to predict DHE output to within 10-15%. The shortcoming of the Culver and Reistad (1978) method is that there is no way of predicting the mixing ratio except by experience.

Pan (1983) examined convection promotion in wells with DHEs for direct application and conducted several field experiments. The model of Culver and Reistad (1978) was applied, and Pan (1983) concluded that the mixing process of water in the well bore was not well understood.

More recently, Chiasson and Gill (2008) applied Kelvin's Line Source Solution to a field-tested DHE in Puna District, Hawaii. That solution introduced a thermal resistance term that essentially lumped all heat transfer processes in the well bore and skin into one parameter. The shortcoming with the Chiasson and Gill (2008) approach was that the Line Source Solution is applicable to heat conduction only, and thus the predicted thermal conductivity value combined conductive and advective heat transport in the aquifer.

The approach used in this present paper for DHE design and predictive output is an analytical solution to the advection-dispersion equation. The solution has been adapted to conductive-advective heat transport for use with borehole heat exchangers by Chiasson and O'Connell (2011). Details are provided in that paper, and are summarized below.

The governing partial differential equation describing mass transport in the subsurface with flowing groundwater is described by the advection-dispersion equation, which has been derived by Bear (1972) and Freeze and Cherry (1979) for contaminant transport. By applying the law of conservation of energy to a control volume, an equation for heat transport can be derived and expressed in two-dimensional Cartesian coordinates. For a homogeneous medium with a uniform velocity and two-dimensional flow with the direction of flow parallel to the x-axis, the governing equation simplifies to:

$$D_L \frac{\partial^2 T}{\partial x^2} + D_T \frac{\partial^2 T}{\partial y^2} - v_x \frac{\partial T}{\partial x} = R \frac{\partial T}{\partial t} \quad (1)$$

where:

$$D_L = a_L v_x + D^* \;\; and \;\; D_T = a_T v_x + D^* \quad (2a,b)$$

A list of symbols is provided in the Nomenclature section at the end of this paper. The $a_L v_x$ and $a_T v_x$ terms are referred to as mechanical dispersion in the longitudinal and transverse directions. In the mass-heat transport analogy, the diffusion coefficient (D^*) is modeled as an effective thermal diffusivity given by:

$$D^* = \frac{k_{eff}}{\emptyset \rho_l c_l} \quad (3)$$

where k_{eff} is defined as $\phi kl + (1\text{-}\phi)ks$, which is a volume-weighted average thermal conductivity of the saturated water/rock matrix and is necessary to distinguish between the thermal conductivity and thermal capacity of the water and soil/rock to account for the fact that heat is stored and conducted through both the water and rock, but heat is only advected by the water. A retardation coefficient (R) is also necessary to adjust the advection and diffusion terms to account for the fact that heat is stored and conducted through both the water and rock, but heat is only advected by the water (Bear, 1972). This is given by:

$$R = 1 + \frac{(1-\emptyset)\rho_s c_s}{\emptyset \rho_l c_l} = \frac{(\rho c)_{eff}}{\emptyset \rho_l c_l} \quad (4)$$

Chiasson and O'Connell (2011) adapted a mass-transport solution to Equation 1 for a continuous injection or extraction of heat (located at the origin, $x = 0$, $y = 0$) into a two-dimensional flow field with uniform groundwater flow velocity (v_x) parallel to the x-axis. The solution assumes an infinite medium with initial temperature T_o, constant thermal conductivity and diffusivity, and with constant heat transfer rate. This solution also assumes that water flows uniformly at constant velocity along the entire borehole length. The boundary conditions are given by:

$$\lim_{r \to 0}\left(-r\frac{\partial T}{\partial r}\right) = \frac{q'}{2\pi k_{eff}} \;\; and \;\; T_{r=\infty} = T_o \quad (5)$$

The solution for ground temperature at time t and distance x and y from the origin, and adjusting for thermal retardation, is given by:

$$\Delta T(x,y,t) = \frac{q'}{4\pi(\rho c)_{eff}\left(\frac{D_L D_T}{R^2}\right)^{1/2}} e^{\frac{v_x x}{2D_L}} \left(\int_{t_D=0}^{t_D=\infty} \frac{e^{-u-B^2/(4u)}}{u} du\right) \quad (6)$$

Chiasson and O'Connell (2011) noted that

$$\int_{\bar{r}^2/4Fo}^{\infty} \frac{e^{-u-b^2/(4u)}}{u} du = W\left(\frac{\bar{r}^2}{4Fo}, b\right)$$

where $W(u,\beta)$ is known in well hydraulics as the leaky well function, and is extensively tabulated by Hantush (1956). Therefore, Equation 6 can be written as:

$$\Delta T(x,y,t) = \frac{q'}{4\pi(\rho c)_{eff}\left(\frac{D_L D_T}{R^2}\right)^{1/2}} e^{\frac{v_x x}{2D_L}} \cdot [W(0,B) - W(t_D,B)] \quad (7)$$

where t_D is a dimensionless form of time given by

$$\left(\frac{v_x}{R}\right)^2 t \Big/ 4\frac{D_L}{R}$$ and $W(0,B) = 2K_0(B)$,

where K_0 is the modified Bessel function of the second kind of order 0. The average borehole wall temperature can be determined by computing temperatures at locations around the borehole wall. Note that for negligible groundwater flow rates, Equation 7 reduces to

$$\Delta T(r,t,\theta) = \frac{q'}{4\pi k_{eff}} W\left(\frac{\bar{r}^2}{4Fo}, 0\right)$$

which is equivalent to Kelvin's infinite line source solution, since

$$W\left(\frac{\bar{r}^2}{4Fo}, 0\right) = W\left(\frac{\bar{r}^2}{4Fo}\right) = \int_{\bar{r}^2/4Fo}^{\infty} \frac{e^{-u}}{u} du$$

where $W(u)$ is known as the well function in well hydraulics.

The average fluid temperature in the DHE (T_f) is then related to the change in the average borehole wall temperature (T_b) through the use of a steady-state borehole thermal resistance per unit length (R'_b):

$$T_f = \Delta T_b + q' \cdot R'_b + T_g \quad (8)$$

METHODOLOGY

Thermal Response Field Testing

A thermal response test was conducted at a residence in Klamath Falls, Oregon on a well that was used directly for space and domestic hot water heating. The DHE configuration consisted of a double PEX u-tube. The well was completed with an was 8-inch (203-mm) diameter casing, approximately 30 ft (9 m) in length, and the well bore depth was recorded on the drilling log as 240 ft (73 m). The static water level in the well was recorded at 100 ft (30.5 m) below grade, giving a submerged length of DHE of approximately 140 ft (42.7 m). The well was completed in a basaltic aquifer.

The thermal response test was conducted using the portable apparatus shown in Figure 1. The undisturbed groundwater temperature was taken as the equilibrated water temperature circulating in the DHE under no thermal load. This temperature was measured at 140˚F (60˚C). A constant heat rate of 3400 W was applied to the fluid stream and the inlet and outlet temperatures to the DHE were recorded at 10-second interval using a Pace Scientific data logger. Raw test data results are shown graphically in Figure 2.

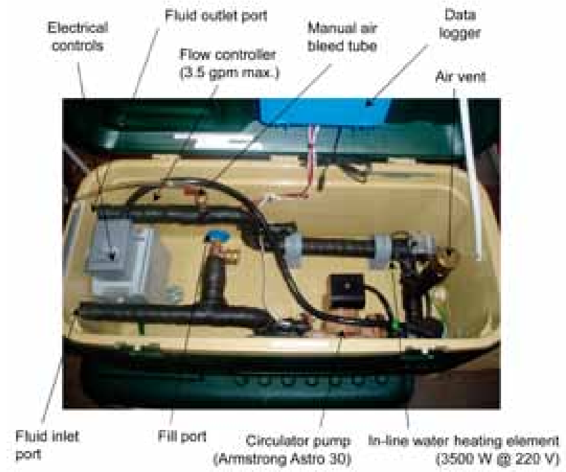

Figure 1: Photograph of portable field-testing apparatus.

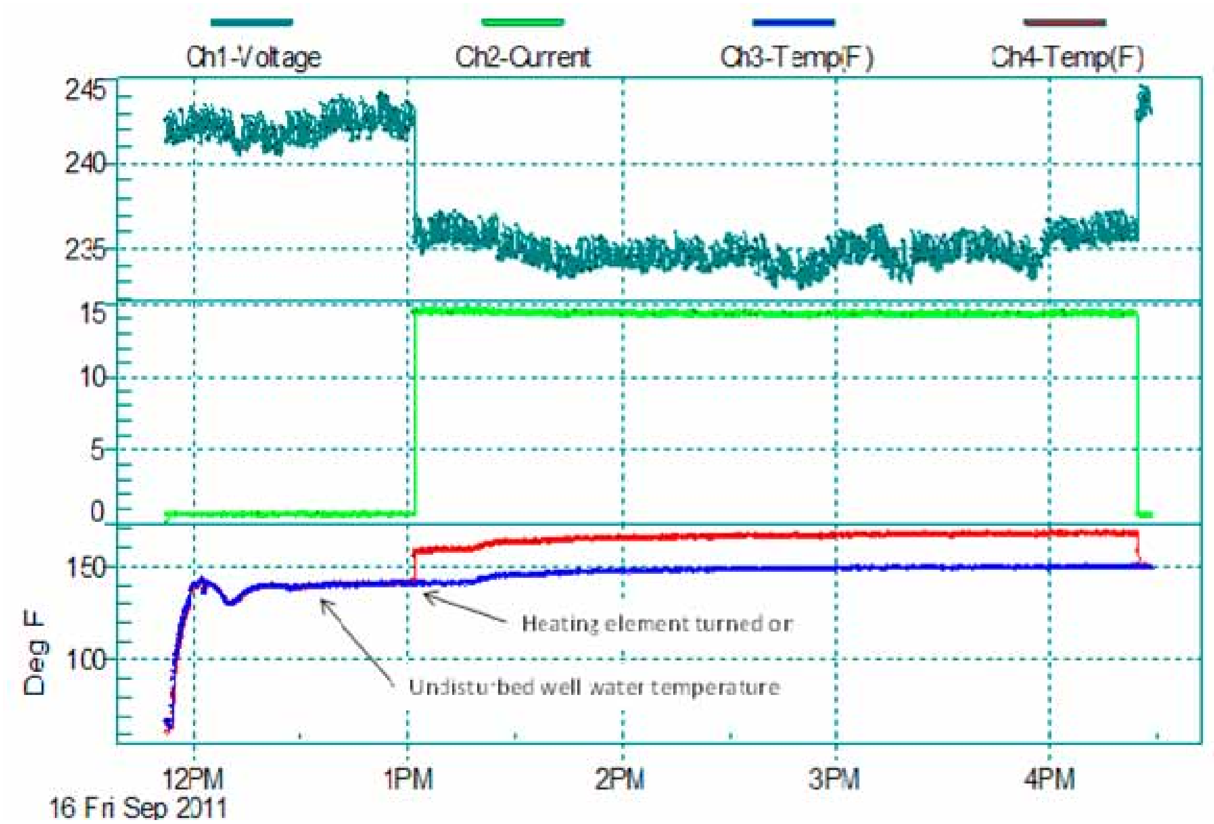

Figure 2: Graph of raw test data showing voltage current from the water heating element. Channel 3 is the water temperature leaving the DHE and Channel 4 is the water temperature entering the DHE. The data sampling rate was 10 seconds.

Application of the Mathematical Model with Parameter Estimation

Application of the analytical solution described above for heat transport in groundwater flow is cumbersome in practice because the groundwater velocity must be known, which requires knowledge of additional parameters, namely hydraulic conductivity, hydraulic gradient, and porosity. The solution using the mass-heat transport analogy requires knowledge of dispersivity, which is very difficult to measure in the field. Consequently, a parameter estimation technique is employed here, as discussed by Chiasson and O'Connell (2011) to determine unknown thermal and hydraulic properties that are relevant to DHE design. The parameters of most interest are: effective thermal conductivity, apparent average linear groundwater velocity, and the borehole thermal resistance. Of secondary interest are the longitudinal and transverse dynamic dispersivity values. Here, the average linear groundwater velocity is described as apparent, because it may not be a true value, given the complex nature of groundwater flow in geothermal aquifers. Therefore, the groundwater velocity may be more appropriately thought of as the effect of groundwater flow on the heat transfer characteristics of the DHE.

Parameter estimation involves minimizing the difference between experimentally obtained results and results predicted by a mathematical model by adjusting inputs to the model. As employed here, the results from the analytical solution are compared to thermal response test results. By systematically varying relevant parameters so that the minimum difference between the experimental results and the mathematical model is attained, a best estimate of the parameters of interest may be found. The relevant parameters varied were k_s, v_x, a_L, a_T, R'_b. An inherent issue with this approach is that the volumetric heat capacity must be estimated because inclusion of it in the optimization results in a non-unique solution. Fortunately, if the rock type is known, volumetric heat capacity does not vary significantly within rock types and does not significantly affect the optimization results.

The objective function for the optimization is the sum of the squared error (SSE) between the numerical model solution and the experimental results at each time of measurement, given by:

$$SSE = \sum_1^N \left(T_{experimental} - T_{model}\right)^2 \quad (9)$$

The optimization is performed with a nonlinear "downhill simplex" optimization technique of Nelder and Mead (1965).

RESULTS AND DISCUSSION

Thermal Response Test Results

Results of the mathematical optimization procedure are as follows:

- Average rock thermal conductivity: 1.2 Btu/hr-ft-F (2.1 W/m-K),
- Average linear groundwater velocity: 5,215 ft/yr (1,590 m/yr),
- Double PEX u-tube DHE thermal resistance (per unit length): 0.129 h-ft-F/Btu (0.0746 m-K/W).

The average rock thermal conductivity is typical of that of volcanic rocks, and the average linear groundwater velocity is of the same order of magnitude determined by tracer tests on the Klamath Falls aquifer. The DHE thermal resistance is similar to that determined in laboratory measurements by Claesson and Hellström (2000).

Crude Model Validation

Heat loss calculations were performed for the residence, and heat rates that the DHE must produce were determined as a function of outdoor air temperature. These heating loads, along with the optimized parameters from the thermal response test, were used as inputs to the analytical solution (Equation 7) to predict DHE output temperatures as a function of outdoor air temperature (Figure 3).

During the first cold spell of the 2011 Fall season in Klamath Falls, the overnight temperature dropped to approximately 35˚F (1.7˚C), and the measured temperature exiting the DHE was 112˚F. As seen from Figure 3, at an outdoor air temperature of 35˚F, the predicted DHE output temperature is 117˚F, which is in excellent agreement with the measured temperature. Obviously, more data are needed to fully validate the model, but initial results are promising.

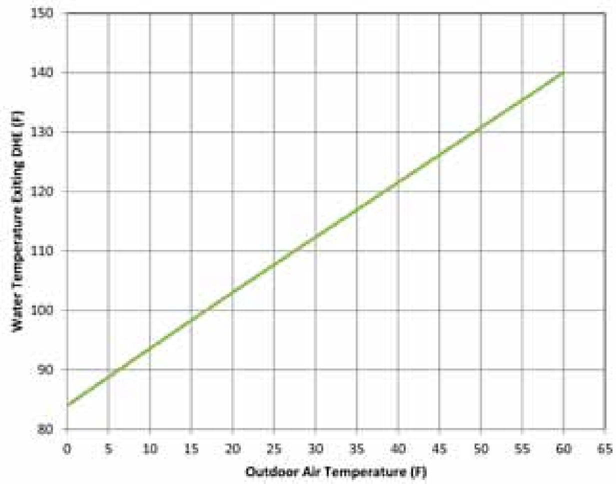

Figure 3: Graph of predicted DHE output temperature as a function of outdoor air temperature.

SUMMARY AND CONCLUSIONS

A useful and powerful method has been presented for determining the thermal output of DHEs in direct applications from geothermal wells. The method includes a readily applied mathematical model with parameters that can be easily measured in the field. With the use of a parameter estimation technique, the method has been roughly, initially validated for a residence in Klamath Falls, Oregon, but further validation of the model is needed.

EDITOR'S NOTE

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NOMENCLATURE

a	dynamic dispersivity (ft [m])
B	$B = [(v_x^2 x^2)/(4D_L^2) + (v_x^2 y^2)/(4D_L D_T)]^{0.5}$
c	specific heat (Btu/lb·°F [J/kg·°C])
D	hydrodynamic dispersion coefficient (ft^2/s [m^2/s])
D^*	effective thermal diffusion coefficient (ft^2/s [m2/s])
H	borehole depth (ft [m])
k	thermal conductivity (Btu/h·ft·°F [W/m·K])
K_0	modified Bessel function of the second kind of order 0
q'	ground thermal load per unit length of vertical bore (Btu/h/ft [W/m])
r	radial distance or radius (ft [m])
R	thermal retardation coefficient (--)
R'_b	borehole effective thermal resistance per unit length of bore (h-ft-°F/Btu [K-m/W])
t	time (s)
T	temperature (°F [°C])
v	average linear groundwater velocity (Ki/ϕ) (ft/s [m/s])
$W(u,\beta)$	Leaky well function (after Hantush, 1956) for arguments u and β
$W(u)$	Well function for argument u (equivalent to the exponential integral)
x, y	distance from origin in Cartesian coordinates

Greek Letters

α	thermal diffusivity (ft^2/h [m^2/s])
ϕ	porosity (--)
ρ	density (lb/ft^3 [kg/m^3])

Subscripts

avg	average	*in*	inlet
b	borehole	*l*	liquid phase
D	dimensionless	*L*	longitudinal
eff	effective	*out*	outlet
f	average fluid	*s*	solid phase
g	undisturbed ground	*T*	transverse
gw	groundwater	*x,y*	coordinate indices